Midterm Exam #2 Formula Sheet

Put-call parity theorem (assuming no dividends; options are European):

$$c + Ke^{-rn\delta t} = p + S,$$

where

$\delta t =$ the length of a timestep; e.g., if the timestep is one year, then $\delta t = 1$.
$n =$ the number of timesteps until expiration;
$c =$ value of a call option with an exercise price of K and n timesteps until expiration;
$p =$ value of a put option with an exercise price of K and n timesteps until expiration;
$r =$ the annualized riskless rate of interest; and
$S =$ the value of the underlying asset.

Risk neutral probability of an "up" move:

$$q = \frac{e^{r\delta t} - d}{u - d},$$

where

$q =$ the risk neutral probability of an "up" move;
$u =$ 1 plus the rate of return from an "up" move; and
$d =$ 1 plus the rate of return from a "down" move.

Risk Neutral Valuation Formula (for a 1 timestep call or put option):

$$f = e^{-r\delta t}[qf_u + (1-q)f_d],$$

where

$f_u =$ the value of the option at the u node; and
$f_d =$ the value of the option at the d node.

Risk Neutral Valuation Formula for an n timestep European call option (AKA the "Cox-Ross-Rubinstein" model):

$$c = e^{-rn\delta t}\left[\sum_{j=a}^{n}\left(\frac{n!}{j!\,(n-j)!}\right) q^j (1-q)^{n-j}\left(u^j d^{n-j} S - K\right)\right],$$

where $a =$ the smallest integer value $> \ln(K/Sd^n)/\ln(u/d)$.

Replicating Portfolio Parameters for call and put options:

$$\text{At the tree's inception, } \Delta = \frac{f_u - f_d}{uS - dS} \text{ and } B = \frac{uf_d - df_u}{e^{r\delta t}(u-d)},$$
$$\text{At node } u,\ \Delta_u = \frac{f_{uu} - f_{ud}}{u^2S - udS} \text{ and } B_u = \frac{uf_{ud} - df_{uu}}{e^{r\delta t}(u-d)}, \text{ and}$$
$$\text{At node } d,\ \Delta_d = \frac{f_{ud} - f_{dd}}{udS - d^2S} \text{ and } B_d = \frac{uf_{dd} - df_{ud}}{e^{r\delta t}(u-d)}.$$

Normal Probability Distribution for the continuously compounded rate of return

$$\ln(S_T/S) \sim N\left((\mu - .5\sigma^2)T, \sigma^2 T\right),$$

where

$\ln(S_T/S) =$ T-period continuously compounded rate of return;
$\mu =$ expected return (per unit of time) in the absence of uncertainty;
$\sigma =$ stock price volatility (per unit of time);
$(\mu - .5\sigma^2)T =$ the mean of $\ln(S_T/S)$; and
$\sigma^2 T =$ the variance of $\ln(S_T/S)$.