

**Wiener Processes**  
**(Finance 4366 Class Problems for 3/26/2020)**

1. A variable,  $x$ , starts at 10 and follows a generalized Wiener process

$$dx = a dt + b dz$$

where  $a = 2$ ,  $b = 3$ , and  $dz$  is a Wiener process.

- (i) What is the mean value of the variable after three years?

**SOLUTION:**  $E(x + dx) = 10 + a dt = 10 + 2(3) = 16$ .

- (ii) What is the standard deviation of the value of the variable after three years?

**SOLUTION:**  $\sigma_x = b\sqrt{dt} = 3\sqrt{3} = 5.19$ .

- (iii) What is the mean value of the variable after six months?

**SOLUTION:**  $E(x + dx) = 10 + a dt = 10 + 2(.5) = 11$ .

- (iv) What is the standard deviation of the value of the variable after six months?

**SOLUTION:**  $\sigma_x = b\sqrt{dt} = 3\sqrt{.5} = 2.12$ .

2. A variable,  $x$ , starts at 10 and follows a generalized Wiener process

$$dx = a dt + b dz$$

During the first two years  $a = 4$  and  $b = 3$ . During the following three years  $a = 6$  and  $b = 4$ .

- (i) What is the mean value of the variable at the end of the five years?

**SOLUTION:**  $E(x + dx) = 10 + a_1 dt_1 + a_2 dt_2 = 10 + 4(2) + 6(3) = 36$ .

- (ii) What is the standard deviation of the variable at the end of the five years?

**SOLUTION:**  $\sigma_x = \sqrt{b_1^2 dt_1 + b_2^2 dt_2} = \sqrt{3^2(2) + 4^2(3)} = 8.12$ .