Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

Options, Futures and Other Derivatives — Name: SOLUTIONS

Dr. Garven

Problem Set 2

In the following problems, assume that returns and risks are annualized.

Problem 1 (50 points). Suppose you are interested in forming a portfolio consisting of two risky securities, securities A and B. Here are their respective return distributions (note: p_s corresponds to the probability that state s occurs, whereas r_{as} and r_{bs} correspond to the state-contingent returns on securities A and B):

p_s	r_{as}	r_{bs}
20%	0%	8%
20%	5%	0%
20%	8%	5%
20%	10%	10%
20%	20%	0%

A. (10 points) What are the expected returns for securities A and B?

SOLUTION: $E(r_a) = \sum_{s=1}^{n} p_s r_{as} = .20(0) + .20(5) + .20(8) + .20(10) + .20(20) = 8.6\%$, and $E(r_b) = \sum_{s=1}^{n} p_s r_{bs} = .20(8) + .20(0) + .20(5) + .20(10) + .20(0) = 4.6\%$.

B. (10 points) What are the standard deviations for the returns on securities A and B?

SOLUTION:

$$\begin{aligned}\sigma_a &= \sqrt{\sum_{s=1}^{n} p_s (r_{as} - E(r_a))^2} \\ &= \sqrt{.20(0-8.6)^2 + .20(5-8.6)^2 + .20(8-8.6)^2 + .20(10-8.6)^2 + .20(0-8.6)^2} = 6.62\%\end{aligned}$$

$$\begin{aligned}\sigma_b &= \sqrt{\sum_{s=1}^{n} p_s (r_{bs} - E(r_b))^2} \\ &= \sqrt{.20(8-4.6)^2 + .20(0-4.6)^2 + .20(5-4.6)^2 + .20(10-4.6)^2 + .20(0-4.6)^2} = 4.08\%\end{aligned}$$

C. (5 points) What is the correlation between returns on securities A and B?

SOLUTION: $\sigma_{ab} = \sum_{s=1}^{n} p_s (r_{as} - E(r_a))(r_{bs} - E(r_b)) = .20\,(0-8.6)\,(8-4.6) + .20\,(5-8.6)\,(0-4.6) + .20\,(8-8.6)\,(5-4.6) + .20\,(10-8.6)\,(10-4.6) + .20\,(20-8.6)\,(0-4.6) = -11.56$; therefore, $\rho_{ab} = \sigma_{ab}/\sigma_a\sigma_b = -11.56/(6.62)(4.08) = -.428$.

D. (10 points) What is the expected return and standard deviation for an equally weighted portfolio consisting of securities A and B?

SOLUTION:

$E(r_p) = w_a E(r_a) + w_b E(r_b) = .5(8.6) + .5(4.6) = 6.6\%$, and $\sigma_p = \sqrt{w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \sigma_{ab}} = \sqrt{.5^2(6.62^2) + .5^2(4.08)^2 + 2(.5)(.5)(-11.56)} = 3.06\%$. Note that since securities A and B are negatively correlated, the standard deviation of an equally weighted portfolio consisting of securities A and B is less than the individual standard deviations for either of these securities. Combining two negatively correlated securities causes individual risks to cancel each other out to a certain extent.

E. (10 points) Suppose securities C and D have the same expected returns and standard deviations as securities A and B, but are uncorrelated. What is the expected return and standard deviation for an equally weighted portfolio consisting of securities C and D?

SOLUTION: The expected rate of return for an equally weighted portfolio consisting of securities C and D is the same as the expected rate of return for an equally weighted portfolio consisting of securities A and B, which is 6.6%. However, the standard deviation for an equally weighted portfolio consisting of securities C and D is higher than it is for an equally weighted portfolio consisting of securities A and B:

$$\sigma_p = \sqrt{w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \sigma_{ab}} = \sqrt{.5^2(6.62^2) + .5^2(4.08)^2 + 2(.5)(.5)(0)} = 3.89\%.$$

F. (5 points) Explain why the portfolio consisting of securities C and D is riskier than the portfolio consisting of securities A and B.

SOLUTION: Since securities C and D are uncorrelated, combinations of C and D provide less diversification benefits than otherwise equivalent combinations of securities A and B; thus, an equally weighted portfolio consisting of securities C and D is riskier than an equally weighted portfolio consisting of securities A and B.

Problem 2 (50 points). Suppose that a security's returns are normally distributed with an expected return of 10% and standard deviation of 20%.

A. (15 points) What is the probability that an investor won't lose money during the coming year if she were to invest in this risky security?

SOLUTION: The probability of losing money by investing in this risky security is $N(z = \frac{0-10}{20} = -.5)$ $= 30.85\%$; thus, the probability of not losing money is $1 - N(-.5) = 69.15\%$.

B. (15 points) Suppose our investor forms a portfolio in which half of her money is allocated to the risky security described in part A and the other half of her money is allocated to a riskless asset with an expected return of 3%. What is the probability she won't lose money during the coming year if she were to invest in this portfolio (hint: the expected return for this portfolio is 6.5%, and its standard deviation is 10%).

SOLUTION: The probability of losing money by investing in this portfolio is $N(z = \frac{0-6.5}{10} = -.65) = 25.78\%$; thus, the probability of not losing money is $1 - N(-.65) = 74.22\%$.

C. (20 points) What is the probability that our investor will earn at least 6% during the coming year if 1) she invests all her money in the risky security described in part A, and 2) she invests all her money in the portfolio described in part B.

SOLUTION: Since 10% is the mean for the risky security, this implies that the probability of earning 6% or more during the coming year by investing all her money in the risky security described in part A is $1 - N(z = \frac{6-10}{20}) = 1 - N(-.2) = 57.93\%$. The probability of earning 6% or more by investing in the portfolio described in part B is $1 - N(z = \frac{6-6.5}{10}) = 1 - N(-.05) = 51.99\%$.