Dividends and American Options

This lecture shows how to extend the binomial pricing model beyond plain-vanilla European puts and calls. We will consider how you allow for dividends and early exercise.

I. Extensions of the Binomial Model

A. Dividend Paying Stocks

B. American Options

I. Extensions of the Binomial Model

A. Dividend Paying Stocks

We can use the binomial model to price options on dividend paying stocks as long as either:

- We know the timing and dollar amount of each dividend to be paid between t and T.
- We know the timing and proportion of the stock to be distributed as dividends between t and T.

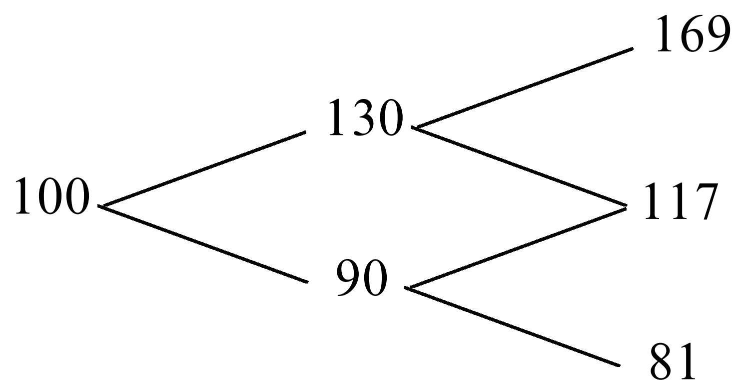
Example:

IBM is currently trading at $\$100$ and the simple quarterly interest rate is 3.33%. Assume that

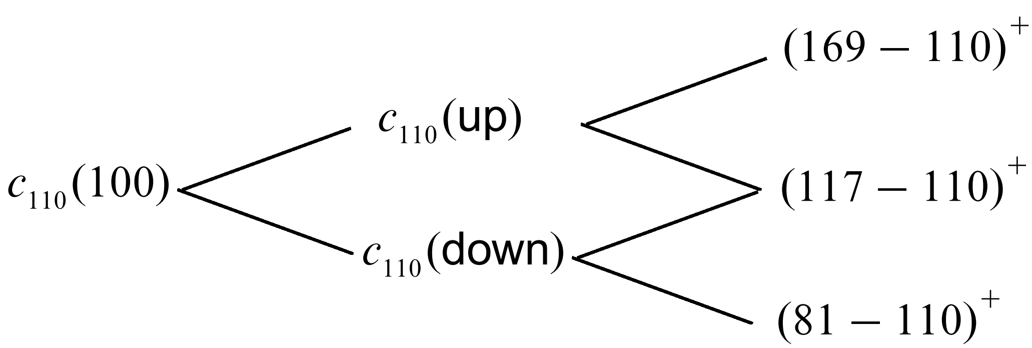
- Every quarter, the price of IBM either
  - rises by 30%,
  - or falls by 10%.
- IBM will pay a 5% dividend in one quarter.

What is the value of a European call with strike price $K = \$110$ and six months to maturity?

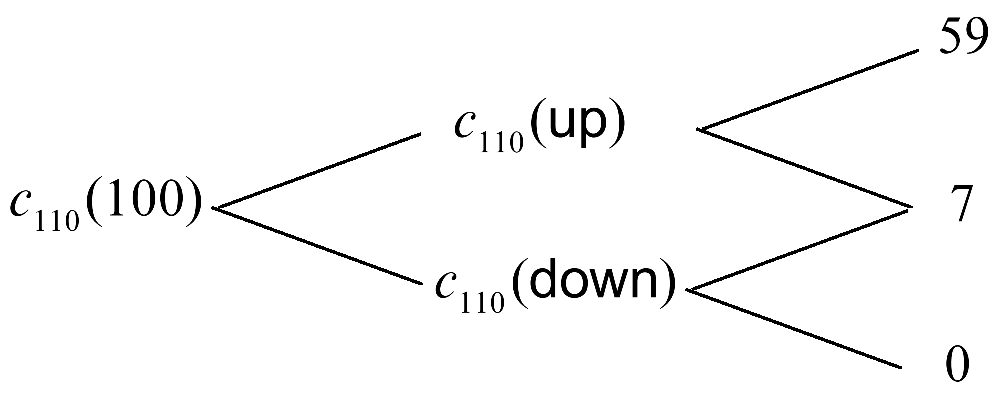
- To get a baseline, let's first consider what would happen if IBM wasn't going to pay a dividend.
  - Then the stocks price tree looks like



- So the price tree for a European call looks like



- That is, the price tree for a European call looks like



- Risk-neutral pricing on the stock implies

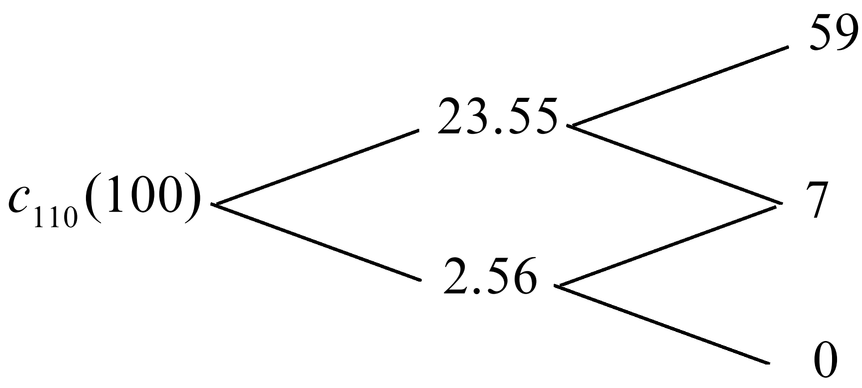
$$q = \frac{1.033 - 0.9}{1.3 - 0.9} = \frac{1}{3},$$

so

$$c_{110}(\text{up}) = \frac{.33 \times 59 + .67 \times 7}{1.033} = 23.55$$

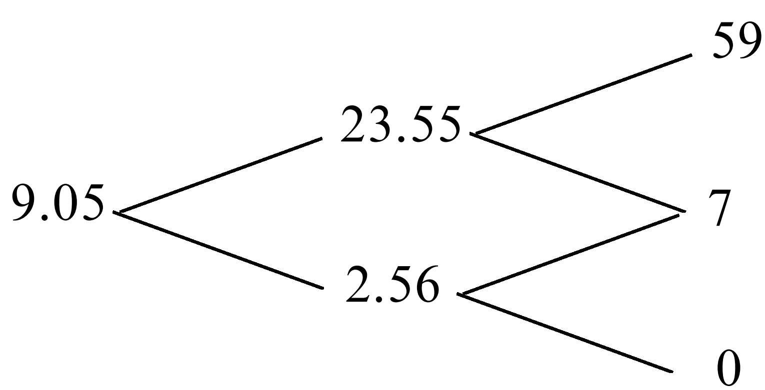
$$c_{110}(\text{down}) = \frac{.33 \times 7 + .67 \times 0}{1.033} = 2.56$$

- So the payoff diagram looks like

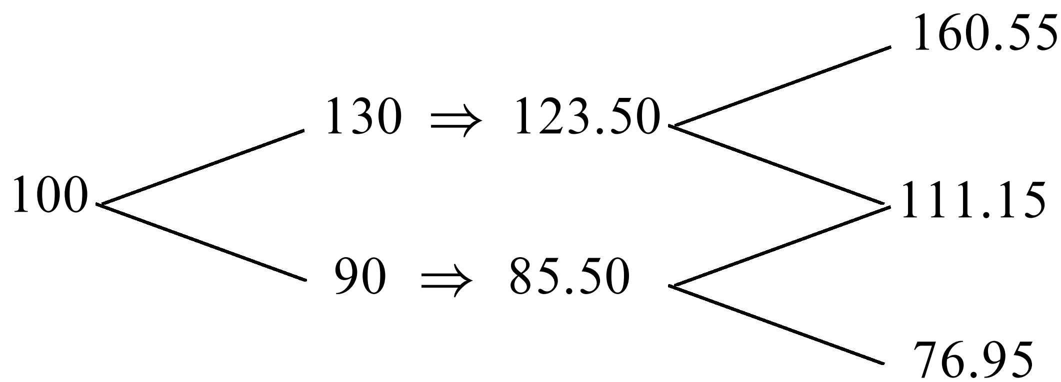


- Finally, the day-zero call price is then

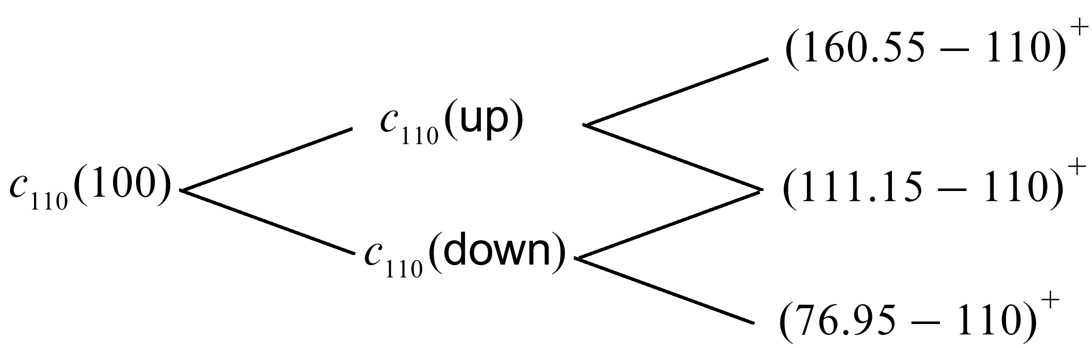
$$c_{110}(100) = \frac{.33 \times 23.55 + .67 \times 2.56}{1.033} = 9.05$$



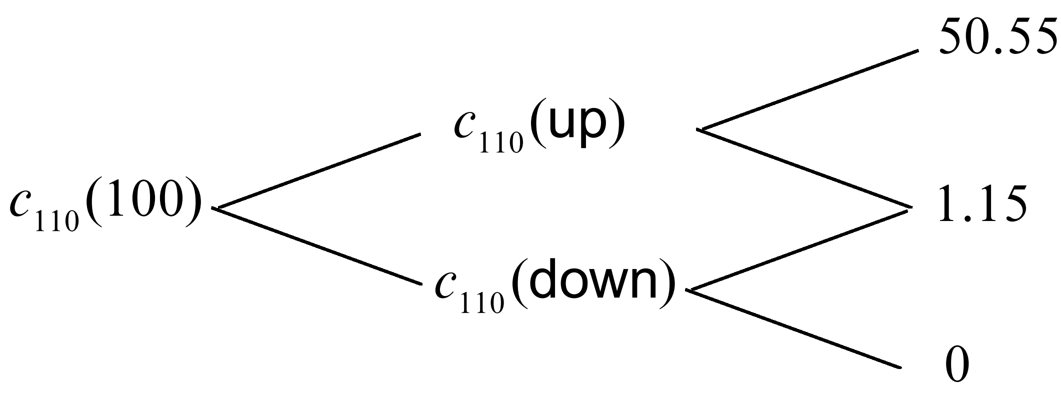
- However, if IBM distributes a 5% dividend at the end of period one, then the stocks price tree looks like



- So the price tree for a European call looks like



- That is, the price tree for a European call looks like



- Risk-neutral pricing on the stock still implies

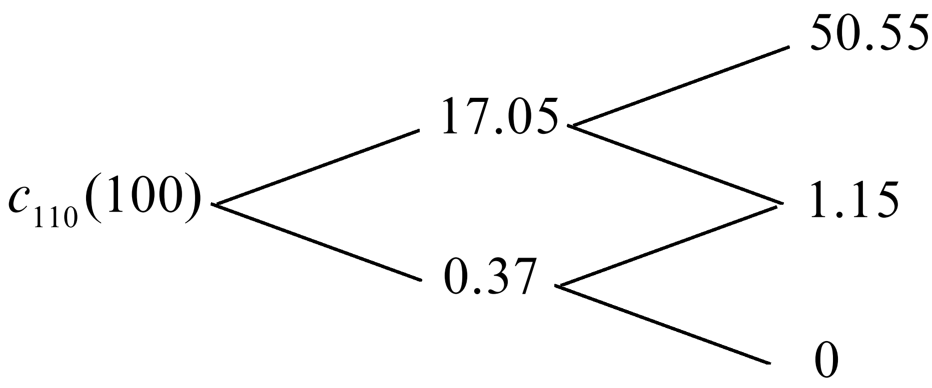
$$q = \frac{1.033 - 0.9}{1.3 - 0.9} = \frac{1}{3},$$

so

$$c_{110}(\text{up}) = \frac{.33 \times 50.55 + .67 \times 1.15}{1.033} = 17.05$$

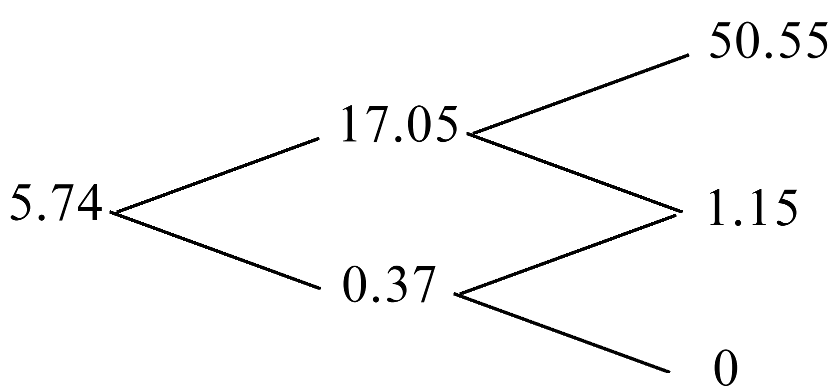
$$c_{110}(\text{down}) = \frac{.33 \times 1.15 + .67 \times 0}{1.033} = 0.37$$

- So the payoff diagram looks like

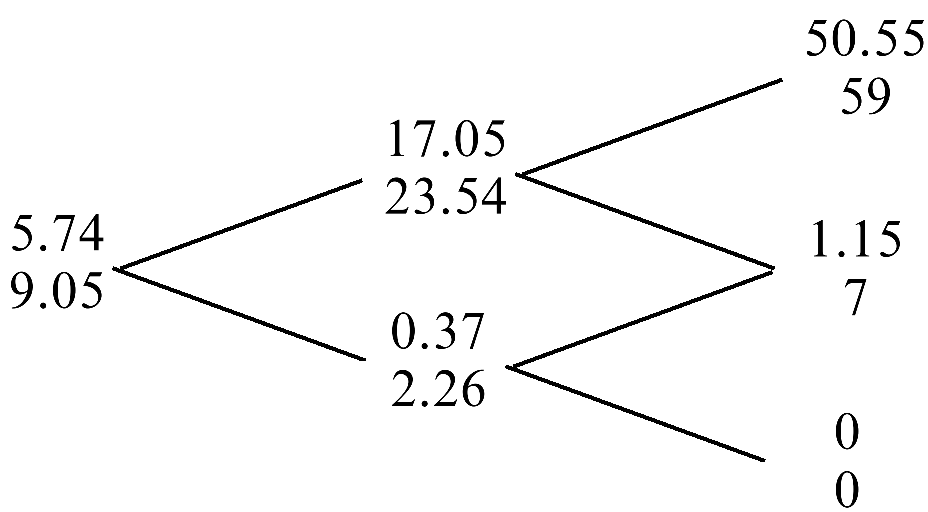


- So the day-zero call price is

$$C_{110}(100) = \frac{.33 \times 17.05 + .67 \times 0.37}{1.033} = 5.74$$



- Let's compare the call on
  1. the dividend paying stock (top)
  2. the non-dividend paying stock (bottom)



- Dividends **reduce** the value of **calls**
- A call is a bet that prices are **going to rise**
- Dividends slow the growth in a stock's price

B. American Options

With the binomial model it is easy to consider the early exercise of an American option.

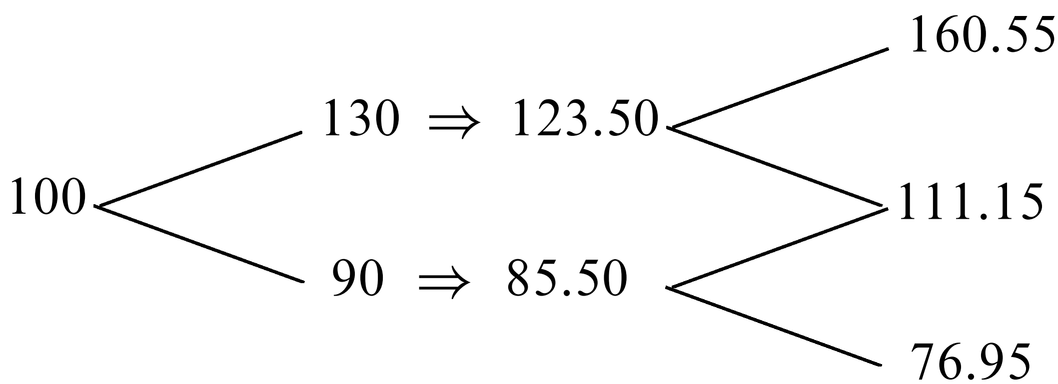
- Just work backwards through the tree, deciding at each node whether to exercise or wait.
  - **Dynamic programming** is SOP
- **Important:** We use
  - The **pre-dividend** price to determine the **exercise value** of a call
  - The **post-dividend** price to determine the **exercise value** of a put

Let's do an example.

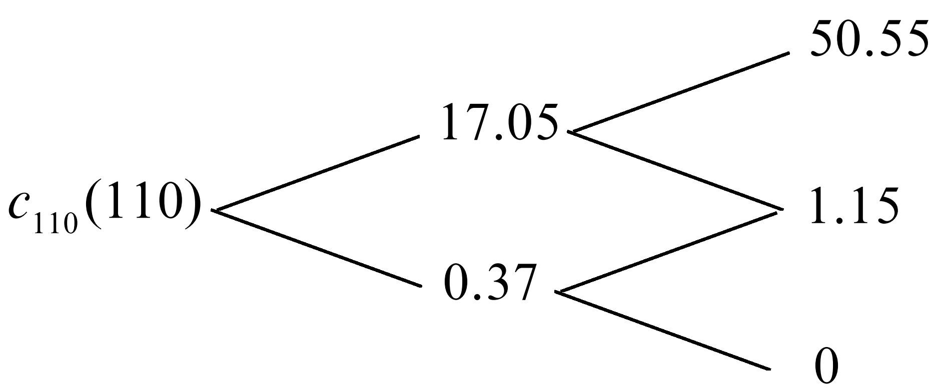
IBM example continued ...

Let's consider an **American** call struck at 110.

- When IBM distributes a 5% dividend at the end of period one, when the tree looks like:



- Here's the payoff diagram for the European call

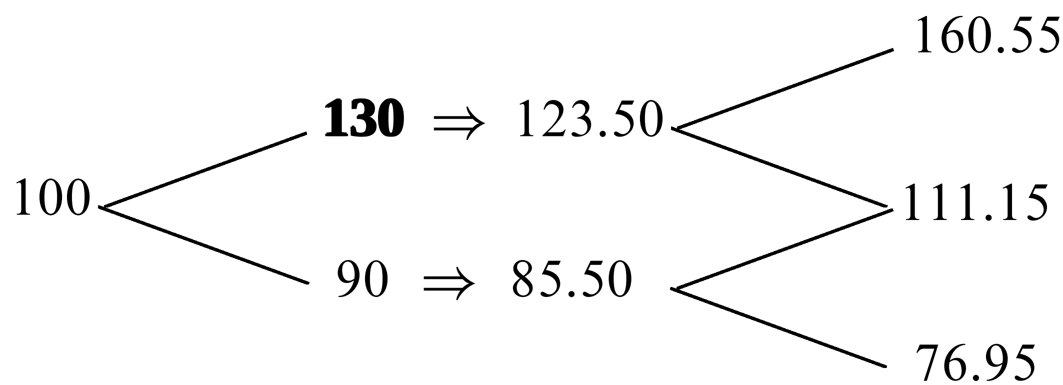


- When would you want to exercise early?
  - When is intrinsic value $>$ continuation value?

- When the European call's "option value" is worth less than the exercise:

  When the stock goes up...

  ... but before the dividend is paid.

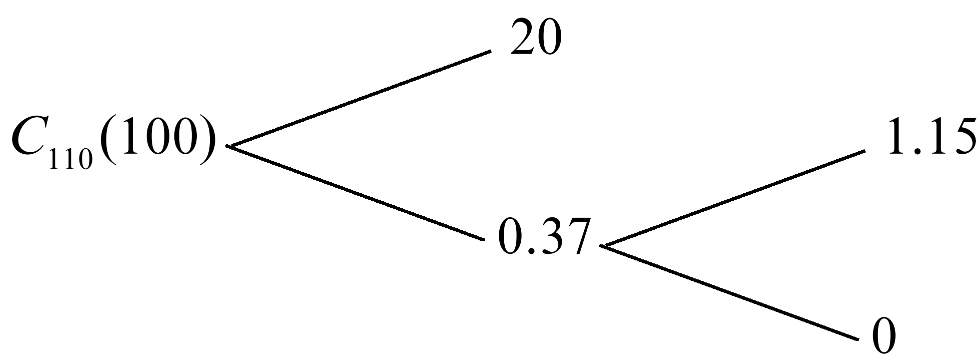


- The value of the European call is then $\$\mathbf{17.05}$...
- ... but if you exercise you get $\$130 - \$110 = \$\mathbf{20}$.

So what's the tree for the American call look like?

Just replace the up-node with the higher value.

- Payoff's for the American call



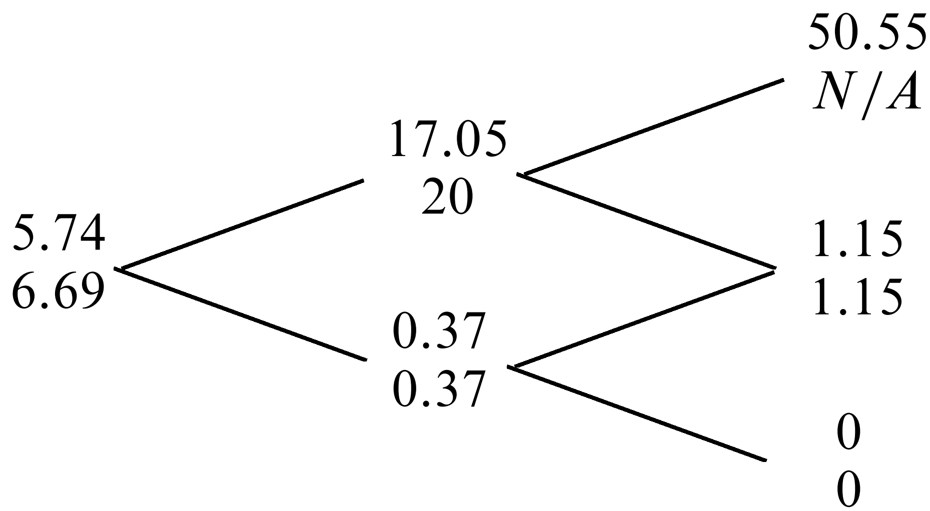
- The day-zero call price is then

$$C_{110}(100) = \frac{.33 \times 20 + .67 \times 0.37}{1.033} = 6.69.$$

- Let's compare the payoff diagrams for

 1. the European call (top)

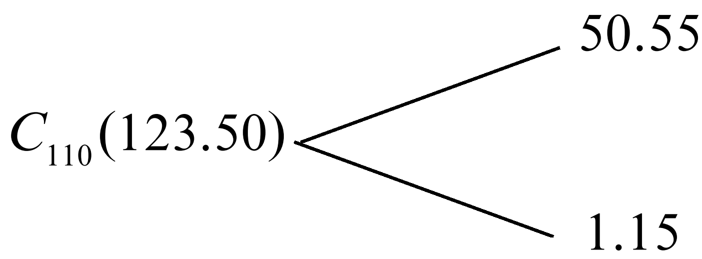
 2. and American call (top)



- The difference, $6.69 - 5.74 = \mathbf{0.95}$, is the value of the **early exercise option.**
  - *I.e.,* it's the value inherent in the additional flexibility provided by the American option.

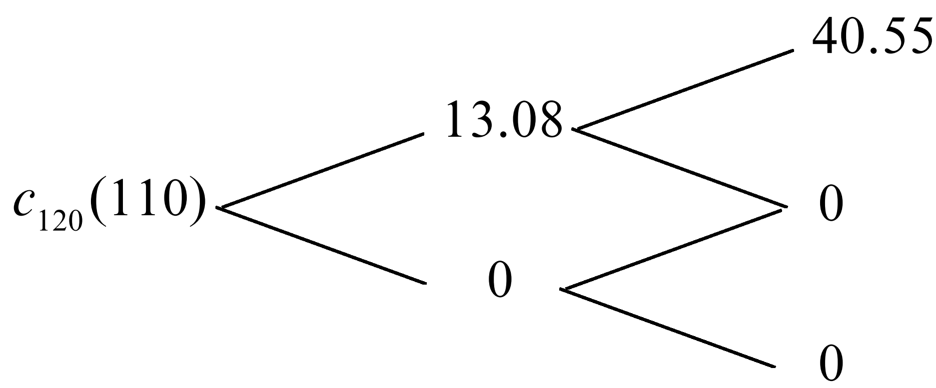
- At what strike would you be (*ex ante*) indifferent between early and late exercise?
  - Is there one?
  - If so, is high or low?

- Let's consider the one-period American call struck at 110, at the "up-node" and *ex dividend*:



- This option is worth $\$17.05$, because you won't exercise early.
  - It's exercise value is only $123.50 - 110 = 13.50$.
- The difference, $17.05 - 13.50 = 3.55$, is the
  - **Interest on the strike:** $(1 - \frac{1}{1.033})110 = 3.55$
  - **The right not to exercise:** $3.55 - 3.55 = 0$
    - In this example the option was guaranteed to finish in the money (it's a forward, effectively)
    - Right not to exercise was worthless.
- Exercise *cum dividend* because $D = 6.5 > 3.55$.
  - *I.e.,* the dividend you capture exceeds the interest you loose and the value of the right not to exercise.

- Then the tree for the call, if you won't exercise, looks like



- Would you still want to exercise early?
  - No! Exercise to capture the dividend, you get

$$130 - 120 = 10.$$

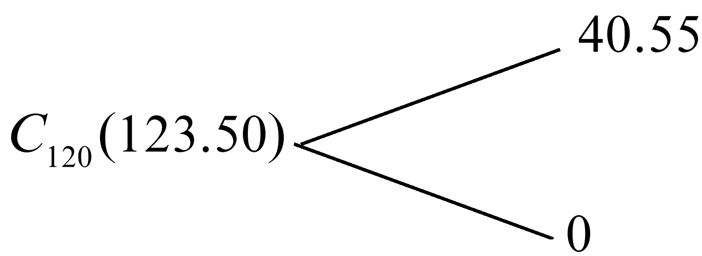
    * And $10 < 13.08$.

- The higher strike makes it **less** likely you'll exercise early.
  - It increases the interest you'll give up (a little).
  - It increases the value of the right not to exercise (potentially a lot).

- That is, you're more likely to exercise DITM.

Dividends and American Options

- Let's consider the one-period American call struck at 120 at the "up-node", *ex dividend*:
  - That's what you get for not exercising
  - What do you get for giving up the dividend?



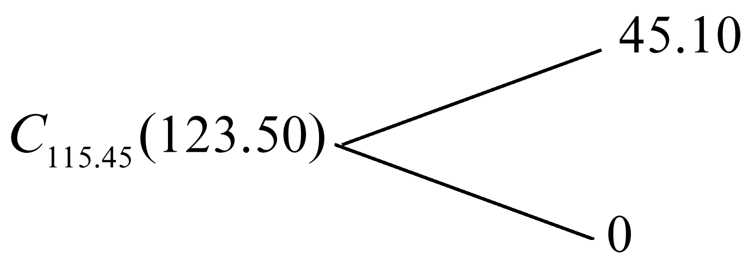
- This option is worth $\frac{40.55/3}{1.033} = 13.08$.
  - It's exercise value is only $123.50 - 120 = 3.50$.

- The difference, $13.08 - 3.50 = 9.58$, is
  - **Interest on the strike:** $(1 - \frac{1}{1.033})120 = 3.87$
  - **The right not to exercise:** $9.58 - 3.87 = 5.71$
    - ∗ In this example the right not to exercise is quite valuable.

- You don't exercise early, because $9.58 > 6.5$.
  - *I.e.,* the dividend you'd capture is less than the interest you'd loose and the value of the right not to exercise.

Dividends and American Options

- We're indifferent if the call is struck at 115.45:



- This option is worth $\frac{45.10/3}{1.033} = 14.55$.
  - Exercise value is only $123.50 - 115.45 = 8.05$.
- The difference is $14.55 - 8.05 = 6.50$
  - Exactly the value of the dividend
  - You can still decompose it:
    * Interest on strike: $(1 - \frac{1}{1.033})115.45 = 3.69$
    * Right not to exercise: $6.50 - 3.69 = 2.81$