Early exercise of American put and call options on a non-dividend paying stock

Question: Should you ever exercise an American put option on a non-dividend paying stock before expiration?

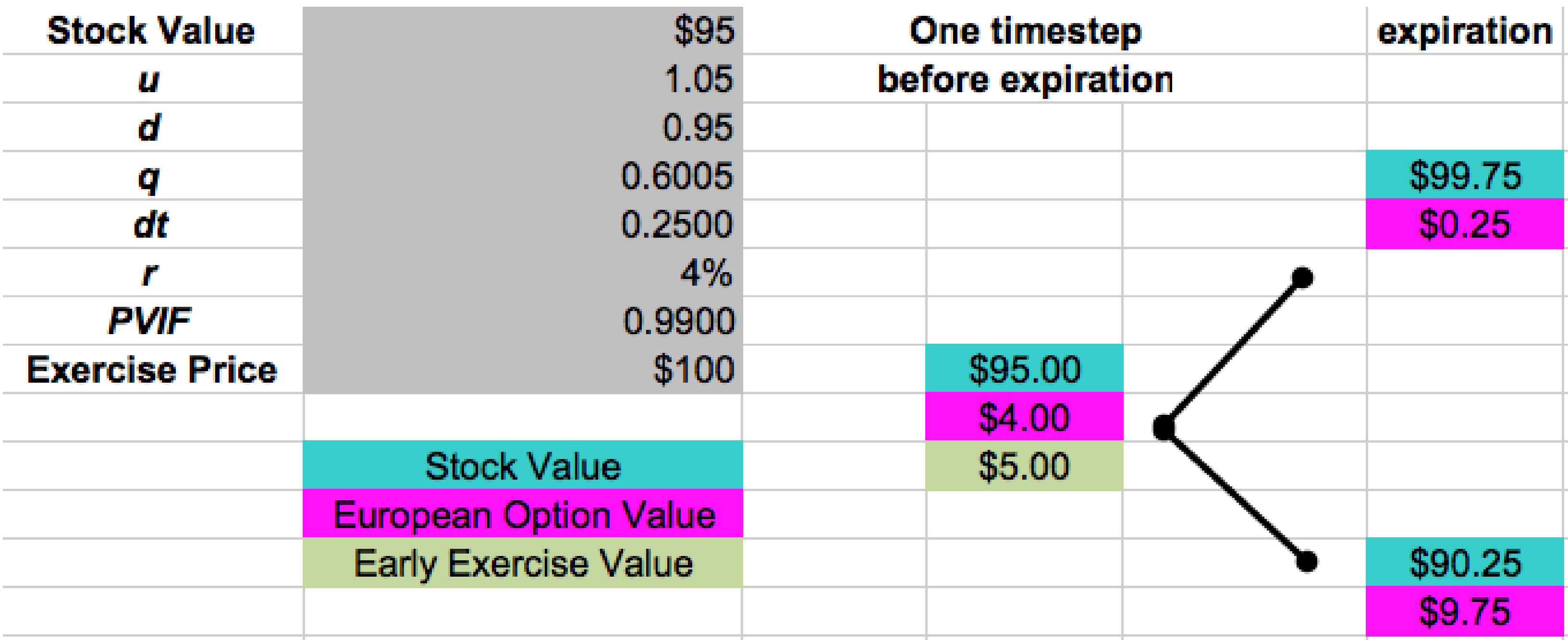
Example: Suppose you own a three-month put struck at \$100. The underlying's price is currently (at $t = 0$) \$95, and the annualized riskless rate of interest is 4%. Should you exercise now (at $t = 0$), or wait until maturity (at $T = .25$)?

- If you *sell* the option at time t you receive $P(S, K, t, T)$, and we know (from page 14 of the 7^{th} lecture in Finance 4366) that $P(S, K, t, T) \geq \max\left[0, Ke^{-r(T-t)} - S(t)\right]$.
- If you *exercise* the option at time t, you receive $K - S(t)$. Therefore, by *exercising* rather than *selling* the option at time t, you earn the following amount:

$$\underbrace{(K - S(t))}_{\text{Proceeds from exercising}} - \underbrace{\left(Ke^{-r(T-t)} - S(t)\right)}_{\text{Proceeds from selling}} = K(1 - e^{-r(T-t)}).$$

By exercising an in-the-money American put early, you effectively earn interest on a bond with a principal value equal to the put's exercise price. In this numerical example, your profit from *exercising* rather than *selling* the option at time t is $\pi = (\$100 - \$95) - (\$100e^{-.01} - \$95) = \$100(1 - e^{-.01}) = \$1$.

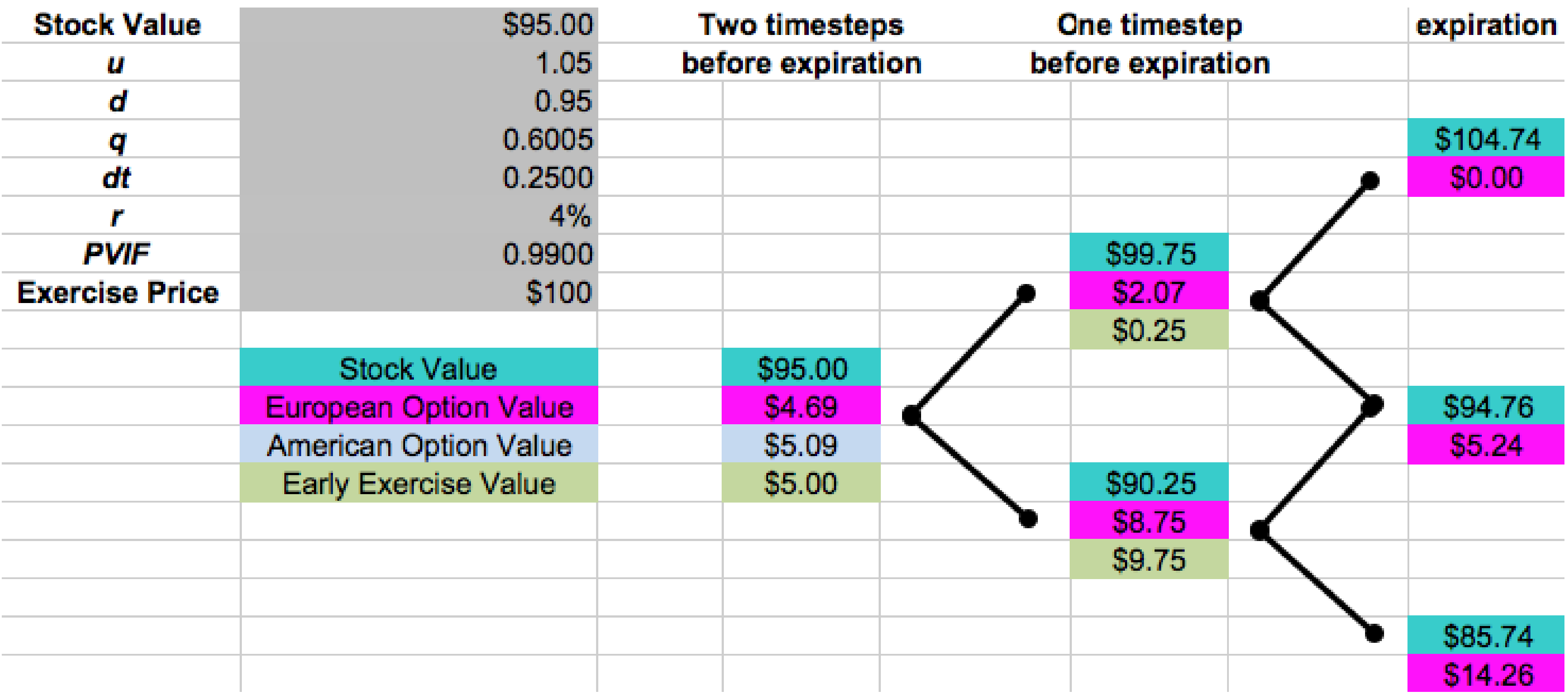
Next, consider the spreadsheet implementation of this numerical example (source: Pricing of American Options on Non-Dividend Paying Stocks spreadsheet):



The various parameters for this numerical example are listed in the upper left portion of this worksheet. Applying risk neutral valuation, we find that the value of the European put is \$4. However, the value of the American put is \$5, which corresponds to its time t

= 0 value based on its early exercise value. Thus, the value of the single-timestep in-the-money American put is greater than the value of an otherwise identical in-the-money European put, and the difference in value corresponds to the interest earned on a bond with a principal value equal to the exercise price.

Next, let's consider an additional three-month timestep:



In this case, early exercise occurs at node *d*. At node *d*, the exercised value of the

American option (\$9.75) exceeds the non-exercised (\$8.75) value of the put by \$1, which corresponds to the interest earned on a bond with a principal value equal to the exercise price. However, at nodes u and the tree's (t =0) inception, the option is more valuable if it is sold rather than exercised. Thus, the value of this in-the-money American put (\$5.09) exceeds the value of an otherwise identical in-the-money European put (\$4.69) by \$.40, since investors anticipate early exercise at node d.

In closing, the answer to the question "Should you ever exercise an American put option on a non-dividend paying stock before expiration?" is a qualified "yes", in the following cases examined here: 1) (sufficiently) in-the-money one timestep American put options and 2) in-the-money and slightly out-of-the-money two timestep American put options.

Question: Should you ever exercise an American call option on a non-dividend paying stock before expiration?

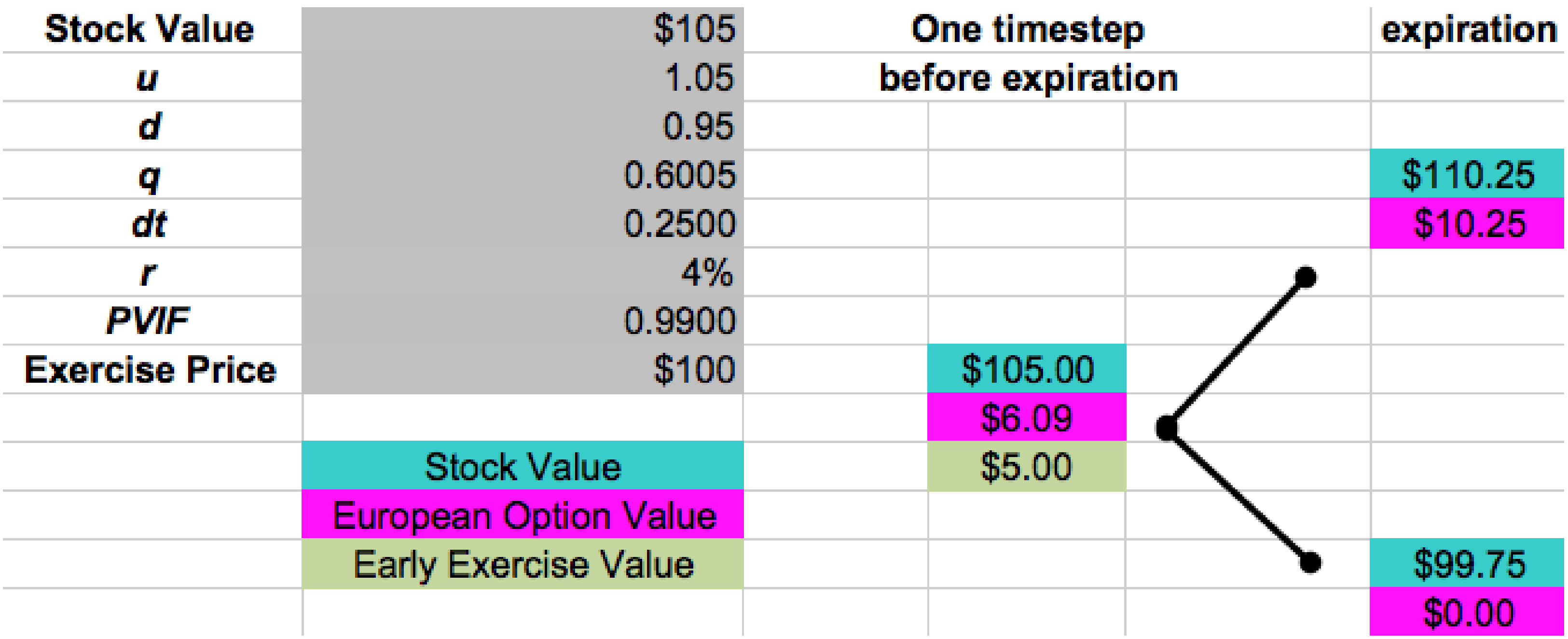
Example: Suppose you own a three-month call struck at \$100. The underlying's price is currently (at $t = 0$) \$105, and the annualized riskless rate of interest is 4%. Should you exercise now (at $t = 0$), or wait until maturity (at $T = .25$)?

- If you *sell* the option at time t you receive $C(S,\ K,\ t,\ T)$, and we know (from the 7^{th} lecture in Finance 4366) that $C\left(S, K, t, T\right) \geq \max\left[0, S(t) - Ke^{-r(T-t)}\right]$.
- If you *exercise* the option at time t, you only receive $S(t) - K$; therefore, by *exercising* rather than *selling* the option at time t, you are guaranteed to lose *at least*

$$\underbrace{(S(t) - K)}_{\text{Proceeds from exercising}} \quad - \quad \underbrace{(S(t) - Ke^{-r(T-t)})}_{\text{Proceeds from selling}} = K(e^{-r(T-t)} - 1).$$

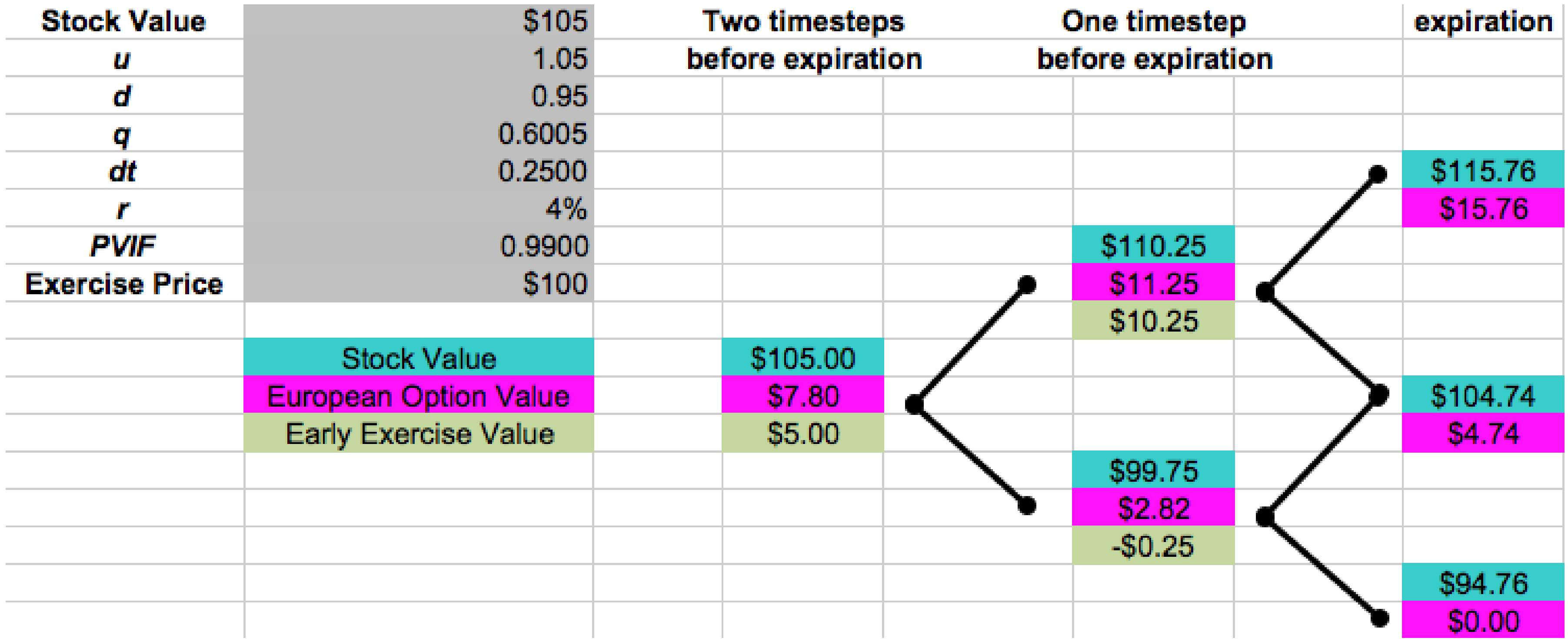
By exercising an in-the-money American call early, you effectively *lose* the opportunity to earn interest on a bond with a principal value equal to the exercise price for the call. In this numerical example, your loss from *exercising* rather than *selling* the option at time t is $\pi = (105 - 100) - (105 - 100e^{-.01}) = 100(e^{-.01} - 1) = -\$1$.

Here is the worksheet for the one timestep American call option (source: Pricing of American Options on Non-Dividend Paying Stocks spreadsheet):



Since the early exercise value of the American call option is less than the value of an otherwise identical in-the-money European call option, it follows that the market value of the American call option is equal to the market value of the European call option.

Adding an additional timestep makes no difference; consider the following worksheet (source: Pricing of American Options on Non-Dividend Paying Stocks spreadsheet):



By inspection, the early exercise value of the American call option at the tree's (t =0) inception as well as nodes u and d is always less than the value of an otherwise identical European call option. In fact, this result trivially generalizes to n timesteps. You *never* want to exercise an American call option early if the underlying doesn't pay dividends, since this means that you will (with certainty) forego the earning of interest.