Early exercise of American put and call options on a non-dividend paying stock

by James R. Garven*
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Question: Should you ever exercise an American put option on a non-dividend paying stock before expiration?

Example: Suppose you own a three-month put struck at \$100. The underlying's price is currently (at $t = 0$) \$95, and the annualized riskless rate of interest is 4%. Should you exercise now (at $t = 0$), or wait until maturity (at $T = .25$)?

- If you *sell* the option at time t you receive $p(S,\ K,\ t,\ T)$, and we know (from page 12 of the 7^{th} lecture in Finance 4366) that $p\left(S, K, t, T\right) \geq \max\left[0, Ke^{-r(T-t)} - S(t)\right]$.[1]

- If you *exercise* the option at time t, you receive $K - S(t)$. Therefore, by *exercising* rather than *selling* the option at time t, you earn the following amount:

$$\underbrace{\left(K - S(t)\right)}_{\text{Proceeds from exercising}} - \underbrace{\left(Ke^{-r(T-t)} - S(t)\right)}_{\text{Proceeds from selling}} = K(1 - e^{-r(T-t)}).$$

By exercising an in-the-money American put early, you effectively earn interest on a bond with a principal value equal to the put's exercise price. In this numerical example, your profit from *exercising* rather than *selling* the option at time t is $\pi = (\$100 - \$95) - (\$100e^{-.01} - \$95) = \$100(1 - e^{-.01}) = \$1$.[2]

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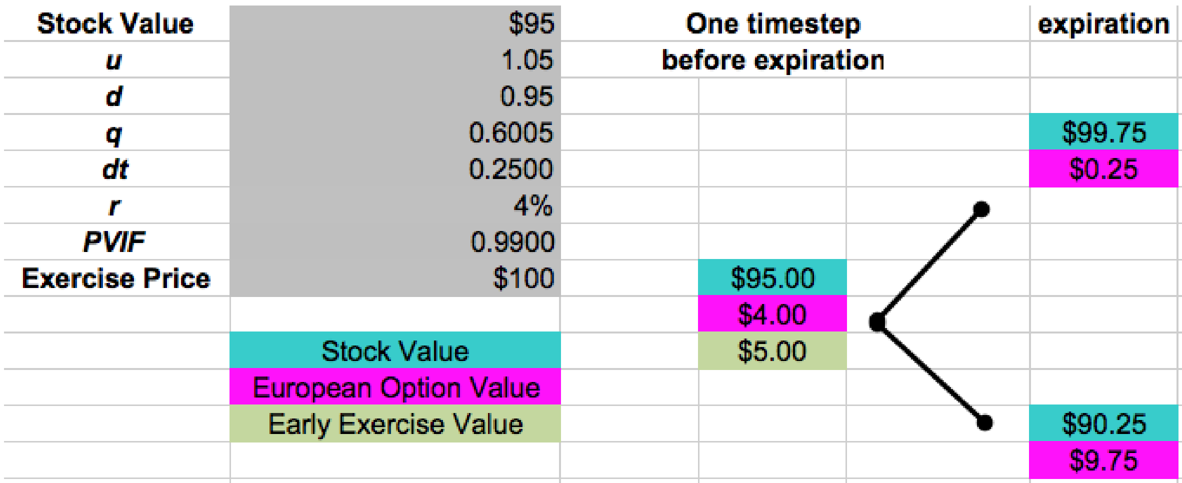
[1] To see *why* this inequality *must* hold, suppose that it doesn't; i.e., suppose the market value of the put (p) is less than its intrinsic value of $p(S,\ K,\ t,\ T) = p(95, 100, 0, .25) = 100e^{-.04(.25)} - 95 = \$4$. Since the put is cheap, a riskless profit of $\pi = 4 - p > 0$ can be earned by 1) buying the put for $\$p$ and the stock for \$95, and 2) shorting the bond for $100e^{-.04(.25)} = \$99$. If the put option subsequently expires in-the-money, the \$100 received by the investor from exercising the put is used to pay off the \$100 owed on the loan. On the other hand, if the put option expires at- or out-of-the-money, then it is worthless, and an even higher riskless profit ($\pi = 4 - p + S(T) - 100$) is earned.

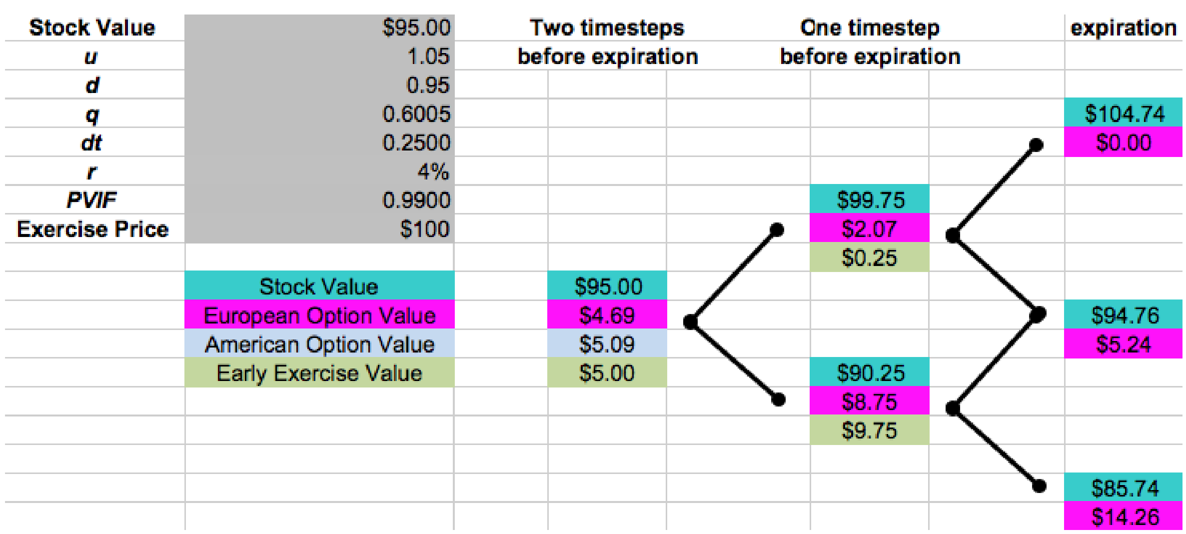
[2] Note that if $r = 0\%$, then there is no advantage to early exercise, which implies that the arbitrage-free value of the American is the same as the the arbitrage-free value of the European put.

Next, consider the spreadsheet implementation of this numerical example (source: Pricing of American Options on Non-Dividend Paying Stocks spreadsheet):



The various parameters for this numerical example are listed in the upper left portion of this worksheet. Applying risk neutral valuation, we find that the value of the European put is \$4. However, the value of the American put is \$5, which corresponds to its time $t = 0$ value based on its early exercise value. Thus, the value of the single-timestep in-the-money American put is greater than the value of an otherwise identical in-the-money European put, and the difference in value corresponds to the interest earned on a bond with a principal value equal to the exercise price.

Next, let's consider an additional three-month timestep:



In this case, early exercise occurs at node d. At node d, the exercised value of the American option (\$9.75) exceeds the non-exercised (\$8.75) value of the put by \$1, which corresponds to the interest earned on a bond with a principal value equal to the exercise price. However, at nodes u and the tree's (t =0) inception, the option is more valuable if it is sold rather than exercised. Thus, the value of this in-the-money American put (\$5.09) exceeds the value of an otherwise identical in-the-money European put (\$4.69) by \$.40, since investors anticipate early exercise at node d.

In closing, the answer to the question "Should you ever exercise an American put option on a non-dividend paying stock before expiration?" is a qualified "yes", in the following cases examined here: 1) (sufficiently) in-the-money one timestep American put options and 2) in-the-money and slightly out-of-the-money two timestep American put options.[3]

Question: Should you ever exercise an American call option on a non-dividend paying stock before expiration?

Example: Suppose you own a three-month call struck at \$100. The underlying's price is currently (at $t = 0$) \$105, and the annualized riskless rate of interest is 4%. Should you exercise now (at $t = 0$), or wait until maturity (at $T = .25$)?

- If you *sell* the option at time t you receive $c(S,\, K,\, t,\, T)$, and we know (from the 7^{th} lecture in Finance 4366) that $c\,(S, K, t, T) \geq \max\left[0, S(t) - Ke^{-r(T-t)}\right]$.[4]

- If you *exercise* the option at time t, you only receive $S(t) - K$; therefore, by *exercising*

[3]Further numerical experimentation using the Pricing of American Options on Non-Dividend Paying Stocks spreadsheet reveals that given the assumed u, d, r, and δt parameter values, early exercise of a one timestep American put option will occur only if the current market value of stock is less than \$96.83. In the case of two timestep American put options, early exercise will occur only if the current market value of stock is less than \$101.93.

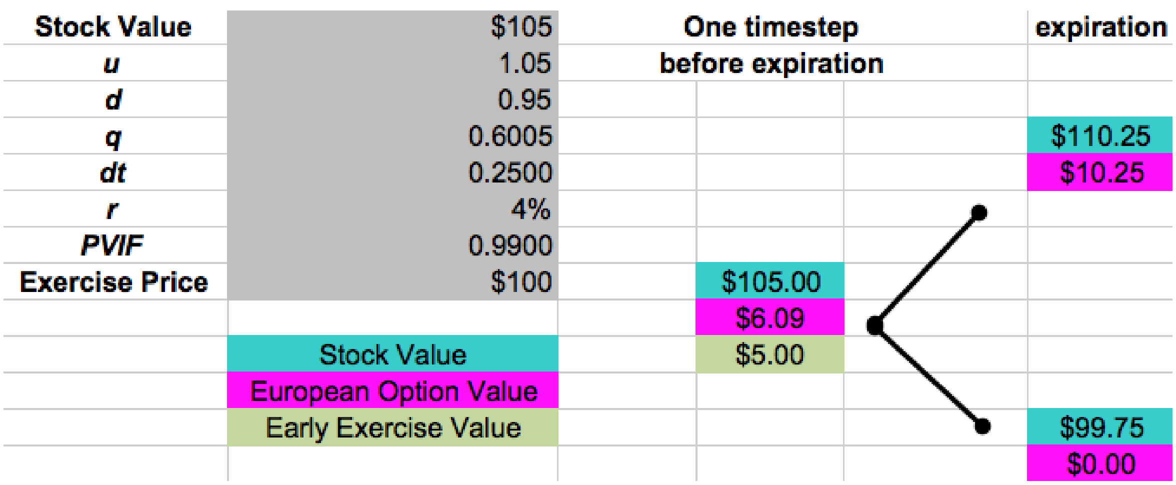
[4]To see *why* this inequality *must* hold, suppose that it doesn't; i.e., suppose the market value of the call (c) is less than its intrinsic value of $c(S, K, t, T) = c(95, 100, 0, .25) = 105 - 100e^{-.04(.25)} = \$6$. Since the call is cheap, a riskless profit of $\pi = 6 - c > 0$ would be earned from 1) buying the call for $\$c$ and the bond for $100e^{-.04(.25)} = \$99$, and 2) shorting the stock for \$105. If the call subsequently expires in-the-money, the \$100 proceeds from the bond investment are used to exercise the call option and cover the short stock position. On the other hand, if the call option expires at- or out-of-the-money, then the call option is worthless, and an even higher riskless profit $\pi = 6 - c + 100 - S(T)$ would be earned.

rather than *selling* the option at time t, you are guaranteed to lose *at least*

$$\underbrace{(S(t)-K)}_{\text{Proceeds from exercising}} \quad - \quad \underbrace{\left(S(t)-Ke^{-r(T-t)}\right)}_{\text{Proceeds from selling}} = K(e^{-r(T-t)}-1).$$

By exercising an in-the-money American call early, you effectively *lose* the opportunity to earn interest on a bond with a principal value equal to the exercise price for the call. In this numerical example, your loss from *exercising* rather than *selling* the option at time t is $\pi = (105-100)-(105-100e^{-.01}) = 100(e^{-.01}-1) = -\$1$.

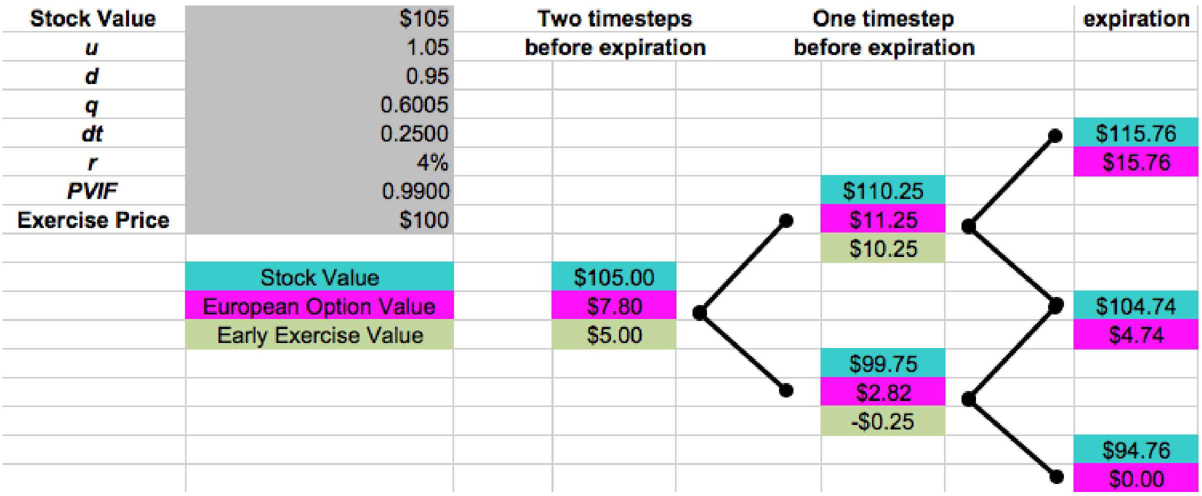
Here is the worksheet for the one timestep American call option (source: Pricing of American Options on Non-Dividend Paying Stocks spreadsheet):



Stock Value	$105	**One timestep**	**expiration**
u	1.05	**before expiration**	
d	0.95		
q	0.6005		$110.25
dt	0.2500		$10.25
r	4%		
PVIF	0.9900		
Exercise Price	$100	$105.00	
		$6.09	
	Stock Value	$5.00	
	European Option Value		
	Early Exercise Value		$99.75
			$0.00

Since the early exercise value of the American call option is less than the value of an otherwise identical in-the-money European call option, it follows that the market value of the American call option is equal to the market value of the European call option.

Adding an additional timestep makes no difference; consider the following worksheet (source: Pricing of American Options on Non-Dividend Paying Stocks spreadsheet):



By inspection, the early exercise value of the American call option at the tree's (t =0) inception as well as nodes u and d is always less than the value of an otherwise identical European call option. In fact, this result trivially generalizes to n timesteps. You *never* want to exercise an American call option early if the underlying doesn't pay dividends, since this means that you will (with certainty) forego the earning of interest.[5]

[5]Furthermore, numerical experimentation with the 1 and 2 timestep call worksheets contained in the Pricing of American Options on Non-Dividend Paying Stocks spreadsheet shows this result (i.e., that you would never want to exercise an American call option early if the underlying doesn't pay dividends) does not depend at all upon the "moneyness" of the call option. The call can be in-, at-, or out-of-the-money and it will always be the case that you don't exercise early.