Mean and Variance of a two-asset portfolio

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The state-contingent return on a two-asset portfolio is equal to the weighted average of the state-contingent returns on the two assets which comprise the portfolio; i.e.,

$$r_{p,s} = w_1 r_{1,s} + w_2 r_{2,s}, \tag{1}$$

where w_1 corresponds to the percent of one's portfolio allocated to asset 1 and $w_2 = 1 - w_1$ corresponds to the percent of one's portfolio allocated to asset 2. Taking expectations of equation (1), we obtain the expected return on the portfolio, $E(r_p)$:

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2). \tag{2}$$

In order to determine the variance of such a portfolio, we need to find the squared deviation of state-contingent portfolio returns (given by equation (1)) from the expected return of the portfolio (given by equation (2)); i.e., $((r_{p,s} - E(r_p))^2$. Substituting the right-hand sides of equations (1) and (2) in place of $r_{p,s}$ and $E(r_p)$ in $(r_{p,s} - E(r_p))^2$, we obtain:

$$\begin{aligned}(r_{p,s} - E(r_p))^2 &= (w_1(r_{1,s} - E(r_1)) + w_2(r_{2,s} - E(r_2))^2 \\ &= w_1^2(r_{1,s} - E(r_1))^2 + w_2^2(r_{2,s} - E(r_2))^2 + 2w_1w_2(r_{1,s} - E(r_1))(r_{2,s} - E(r_2)). \end{aligned} \tag{3}$$

Since $\sigma_p^2 = E(r_{p,s} - E(r_p))^2$, we obtain the formula for the variance of a two-asset portfolio by taking expectations of both sides of equation (3):

$$\begin{aligned}\sigma_p^2 &= w_1^2 E(r_{1,s} - E(r_1))^2 + w_2^2 E(r_{2,s} - E(r_2))^2 + 2w_1w_2 E(r_{1,s} - E(r_1))(r_{2,s} - E(r_2)) \\ &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}. \end{aligned} \tag{4}$$

Thus, the variance of a two-asset portfolio is equal to the weighted average (with weights squared) of the individual asset variances (i.e., $w_1^2\sigma_1^2 + w_2^2\sigma_2^2$) plus 2 times the covariance between the two assets, multiplied by the product of the weights for asset 1 and asset 2 (i.e., $2w_1w_2\sigma_{12}$). The sum given by $w_1^2\sigma_1^2 + w_2^2\sigma_2^2$ captures the contribution to portfolio variance of the individual asset variances, whereas the $2w_1w_2\sigma_{12}$ term captures the contribution to portfolio variance of the manner in which the returns on the two assets covary with each other.[1]

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[1]Note that if $\sigma_{12} = 0$, then portfolio variance depends solely on individual asset variances and the weighting scheme chosen by the investor. If $\sigma_{12} > 0$ ($\sigma_{12} < 0$), then portfolio variance increases (decreases) relative to the zero covariance benchmark. Finally, if $\sigma_{12} = \sigma_1\sigma_2$, then $\sigma_p = w_1\sigma_1 + w_2\sigma_2$; i.e., no diversification occurs.