# Baylor University
# Hankamer School of Business
# Department of Finance, Insurance & Real Estate

Options, Futures and Other Derivatives — Name: SOLUTIONS
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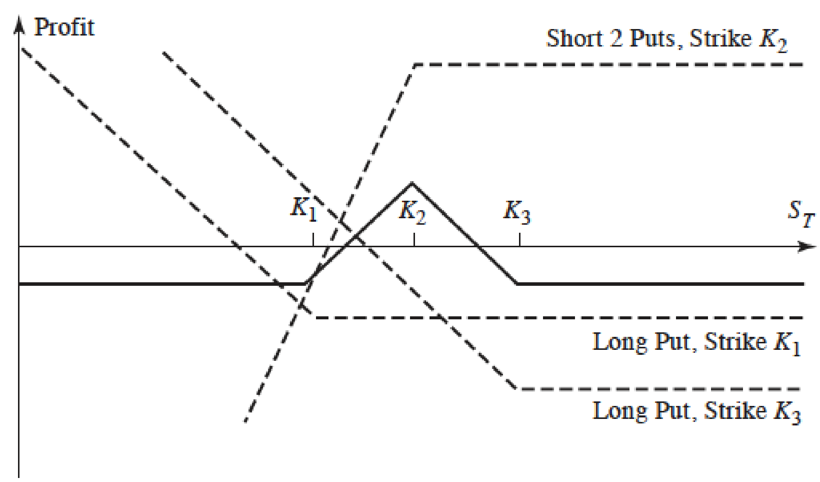
Problem Set 5

**Problem 1 (40 points).**
Three put options on a stock have the same expiration date and strike prices of \$55, \$60, and \$65. The market prices are \$3, \$5, and \$8, respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy. For what range of stock prices would the butterfly spread lead to a loss?

SOLUTION: A butterfly spread is created by buying the \$55 put, buying the \$65 put and selling two of the \$60 puts. This costs $3+8-2\times 5 = \$1$ initially. The following table shows the profit/loss from the strategy.

| *Stock Price* | *Payoff* | *Profit* |
|---|---|---|
| $S_T \geq 65$ | 0 | $-1$ |
| $60 \leq S_T < 65$ | $65 - S_T$ | $64 - S_T$ |
| $55 \leq S_T < 60$ | $S_T - 55$ | $S_T - 56$ |
| $S_T < 55$ | 0 | $-1$ |

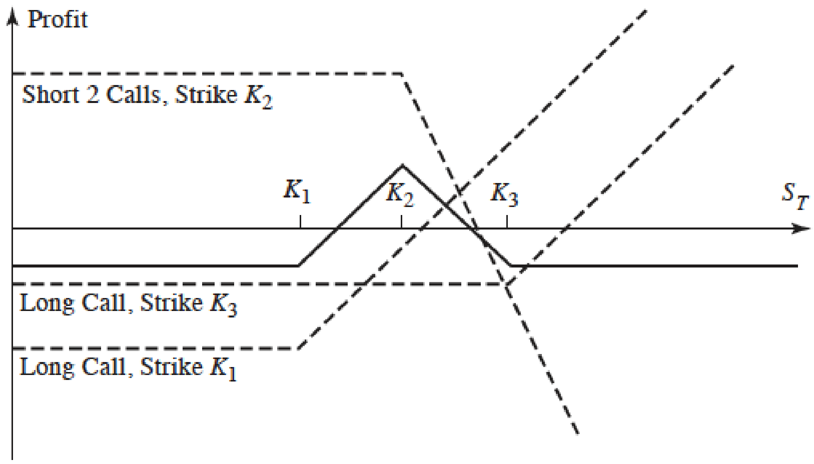
The butterfly spread leads to a loss when the final stock price is greater than \$64 or less than \$56; the source of loss is the initial net cost of \$1 incurred in setting up the butterfly spread strategy, since date $T$ payoffs on the spread itself are non-negative. The (net of setup cost) payoffs shown in the table above resemble the following payoff diagram:




**Problem 2 (40 points).**
Suppose that $c_1$, $c_2$, and $c_3$ are the prices of European call options with exercise prices $K_1$, $K_2$, and $K_3$, respectively, where $K_3 > K_2 > K_1$ and $K_3 - K_2 = K_2 - K_1$. All options have the same time to expiration. With these options, form a Date $T$ Portfolio that is long one option with exercise price $K_1$, long one option with exercise price $K_3$, and short two options with exercise price $K_2$.

1. Identify and explain what kind of strategy this is (hint: the possible choices include bull, bear, butterfly, or calendar).

   SOLUTION: This is a butterfly spread; the following (net of setup cost) payoff diagram shows how this trading strategy resembles the put-based trading strategy in Problem 1; only here, a similar overall effect obtains from employing call options rather than put options:




2. Show that the price of the call option with exercise price $K_2$ (i.e., $c_2$) must be less than or equal to half of the sum of the prices of the call options with the $K_1$ and $K_3$ exercise prices (i.e., $c_1$ and $c_3$); i.e., show that $c_2 \leq 0.5(c_1 + c_3)$.

   SOLUTION: We start by showing that date $T$ payoffs on this butterfly spread are non-negative over the $S_T \leq K_1, K_1 < S_T \leq K_2, K_2 < S_T \leq K_3$, and $S_T > K_3$ terminal share price intervals:

   $S_T \leq K_1 \Rightarrow$ Date $T$ Payoff $= 0$

   $K_1 < S_T \leq K_2 \Rightarrow$ Date $T$ Payoff $= S_T - K_1$

   $K_2 < S_T \leq K_3 \Rightarrow$ Date $T$ Payoff $= S_T - K_1 - 2(S_T - K_2) = K_2 - K_1 - (S_T - K_2) \geq 0$

   $S_T > K_3 \Rightarrow$ Date $T$ Payoff $= S_T - K_1 - 2(S_T - K_2) + S_T - K_3 = K_2 - K_1 - (K_3 - K_2) = 0$

   The payoffs on this butterfly spread are non-negative; i.e., they are always either positive or zero when the options which comprise this spread expire at date $T$. Since this is true, it must also be true that the current market value of the spread, $c_1 + c_3 - 2c_2 \geq 0$; otherwise an arbitrage opportunity would exist. Solving for $c_2$, we find that $c_2 \leq 0.5(c_1 + c_3)$.

**Problem 3 (20 points).**
Draw a picture of trading profits and losses which obtain from selling a call option with an exercise price of $K_2$ and buying a put option with an exercise price of $K_1$. Assume that both options share the same expiration date, and $K_2 > K_1$. What type of forward position obtains when both of these options also share the same exercise price (i.e., $K_1 = K_2$)?

SOLUTION:The trading profits and losses which obtain from selling a call option with an exercise price of $K_2$ and buying a put option with an exercise price of $K_1$ are shown in the diagram below.

The position formed here is commonly referred to as a "range forward"; when both of these options also share the same exercise price (i.e., when $K_1 = K_2$), then this Date $T$ Portfolio is synthetically equivalent to a short forward.