Effects of Dividends on the Pricing of European and American Options

March 15, 2024

This lecture note further considers the extension of the binomial pricing model beyond plain-vanilla European puts and calls. We consider how dividend payments on the underlying asset affect European options' arbitrage-free prices and create incentives for the early exercise of American call options.

I. Extensions of the Binomial Model
   A. Dividend Paying stocks
   B. American Options

Dividends and American Options

I. Extensions of the Binomial Model

A. Dividend Paying stocks

We can use the binomial model to price options on dividend-paying stocks as long as either:

- We know the timing and dollar amount of each dividend to be paid between t and T.
- We know the timing and proportion of the stock to be distributed as dividends between t and T.

Example:

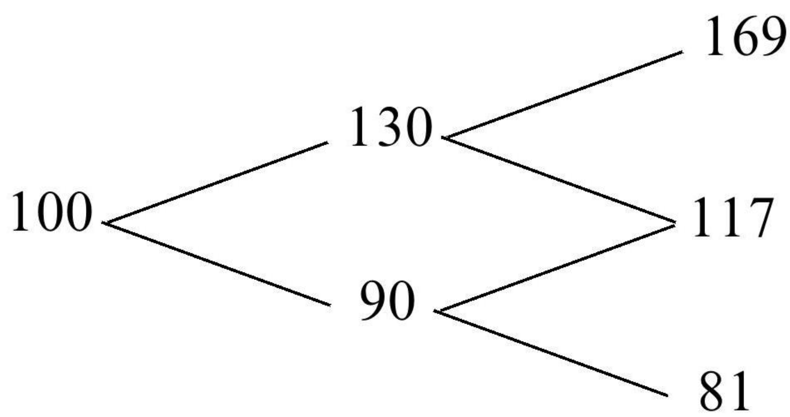
ABC is currently trading at \$100 and the discrete quarterly interest rate is 3.33%. Assume that

- Every quarter, the price of ABC either
- rises by 30%,
- or falls by 10%.

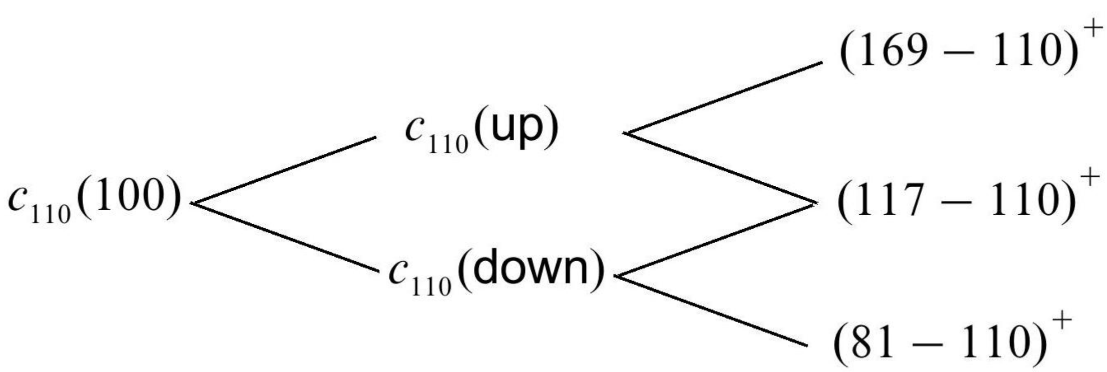
- ABC will pay a 5% dividend in one quarter.

What is the value of a European call with exercise price price $K = \$110$ and six months to maturity?

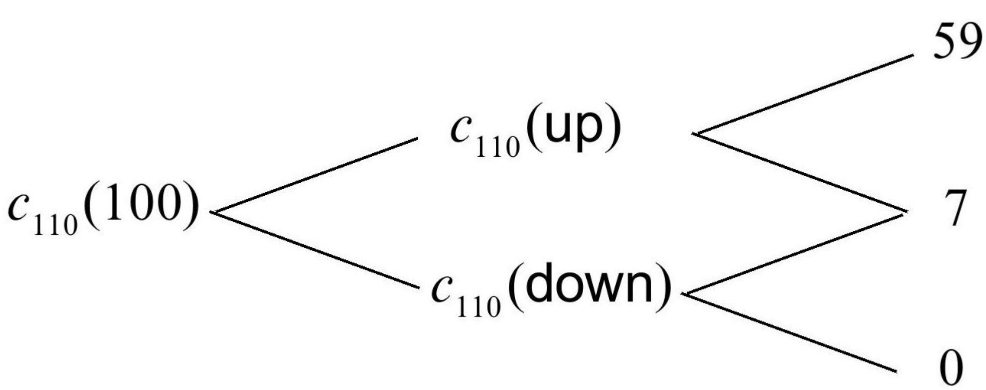
- To establish a baseline, let's first consider what would happen if ABC wasn't going to pay a dividend.
- Then the stock's price tree looks like



- So the price tree for a European call looks like



- That is, the price tree for a European call looks like



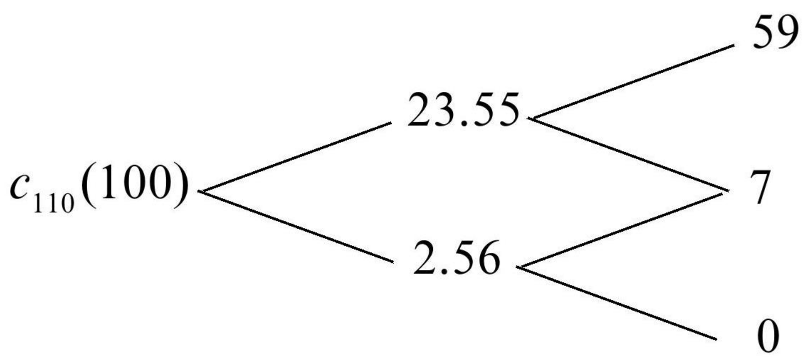
- Risk-neutral pricing on the stock implies

$$q = \frac{1.033 - 0.9}{1.3 - 0.9} = \frac{1}{3}$$

so

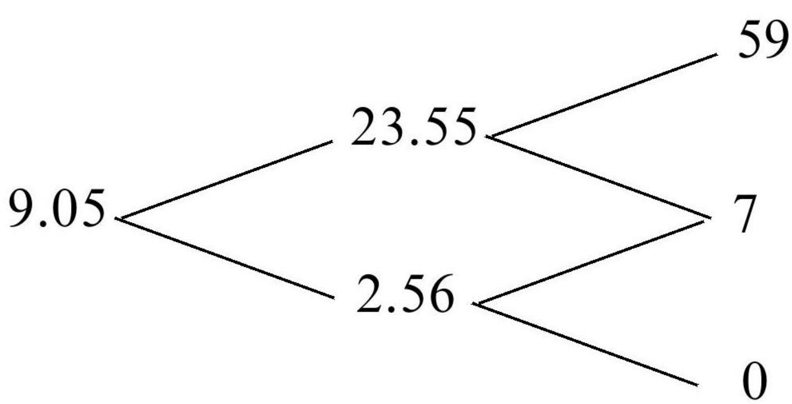
$$c_{110}(\text{ up }) = \frac{.33 \times 59 + .67 \times 7}{1.033} = 23.55$$
$$c_{110}(\text{ down }) = \frac{.33 \times 7 + .67 \times 0}{1.033} = 2.56$$

- So the payoff diagram looks like

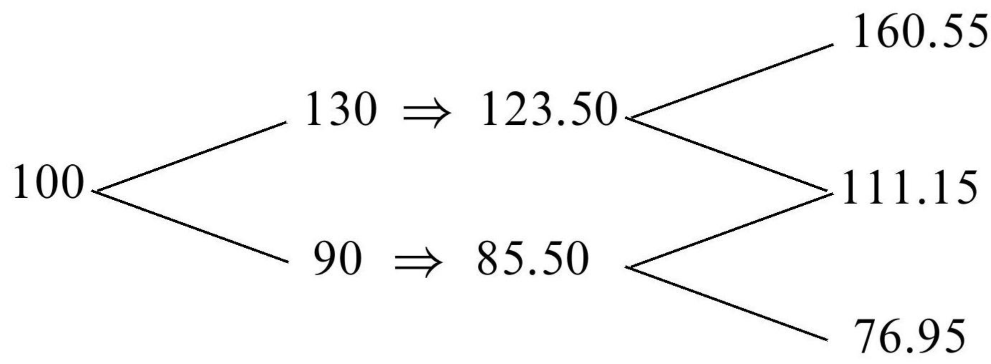


- Finally, the day-zero call price is then

$$c_{110}(100) = \frac{.33 \times 23.55 + .67 \times 2.56}{1.033} = 9.05$$



- However, if ABC distributes a 5% dividend at the end of period one, then the stock's price tree looks like



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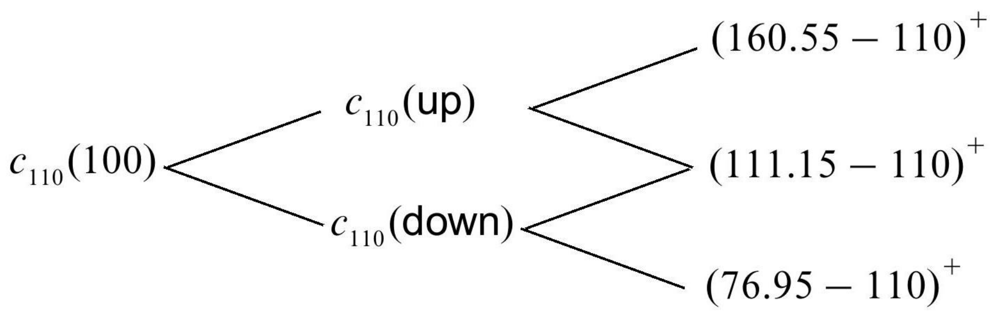

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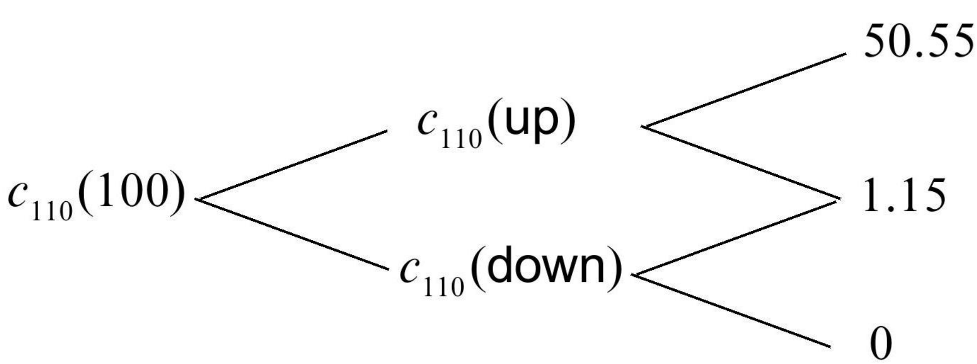


- Risk-neutral pricing on the stock still implies

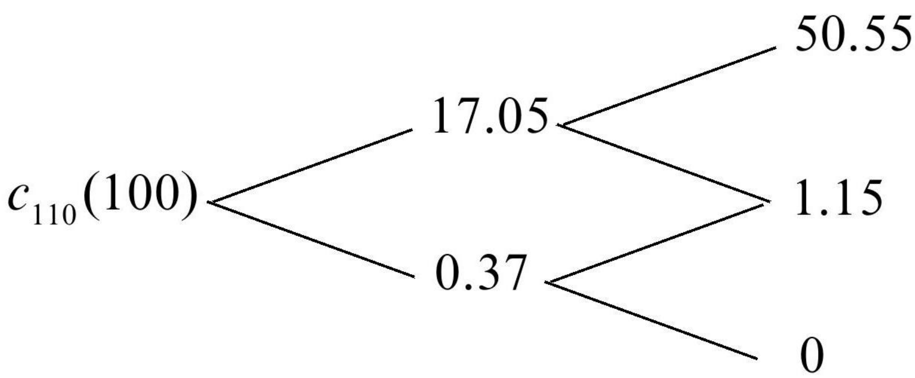
$$q = \frac{1.033 - 0.9}{1.3 - 0.9} = \frac{1}{3},$$

so

$$c_{110}(\text{ up }) = \frac{.33 \times 50.55 + .67 \times 1.15}{1.033} = 17.05$$

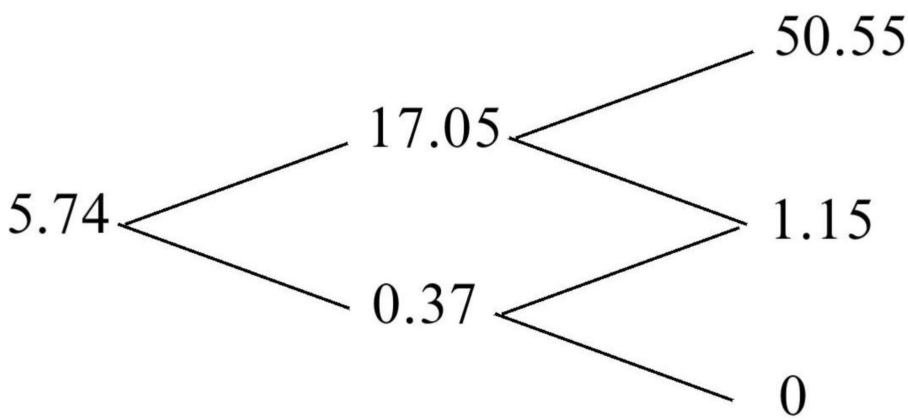
$$c_{110}(\text{ down }) = \frac{.33 \times 1.15 + .67 \times 0}{1.033} = 0.37$$

- So the payoff diagram looks like



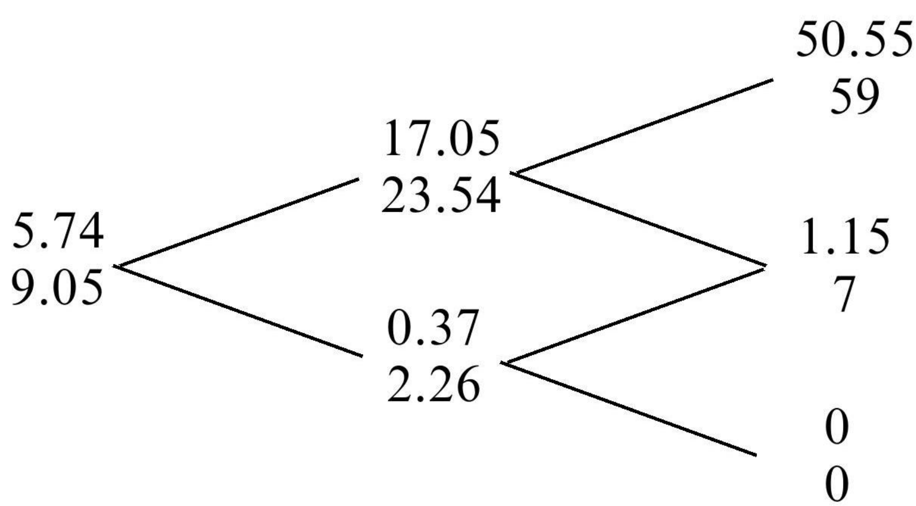
- So the day-zero call price is

$$C_{110}(100) = \frac{.33 \times 17.05 + .67 \times 0.37}{1.033} = 5.74$$



- Let's compare the call on

1. the dividend paying stock (top)
2. the non-dividend paying stock (bottom)



- Dividends reduce the value of calls;
- A call is a bet that prices are going to rise; and
- Dividends slow the growth in a stock's price over time.

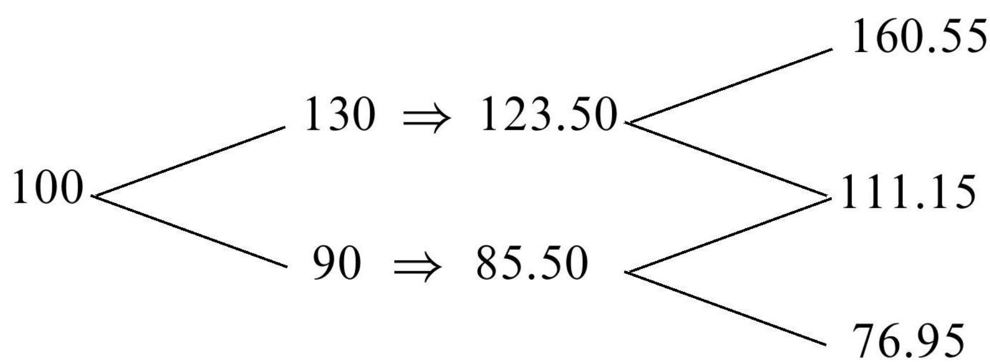
B. American Options

With the binomial model it is easy to consider the early exercise of an American option.

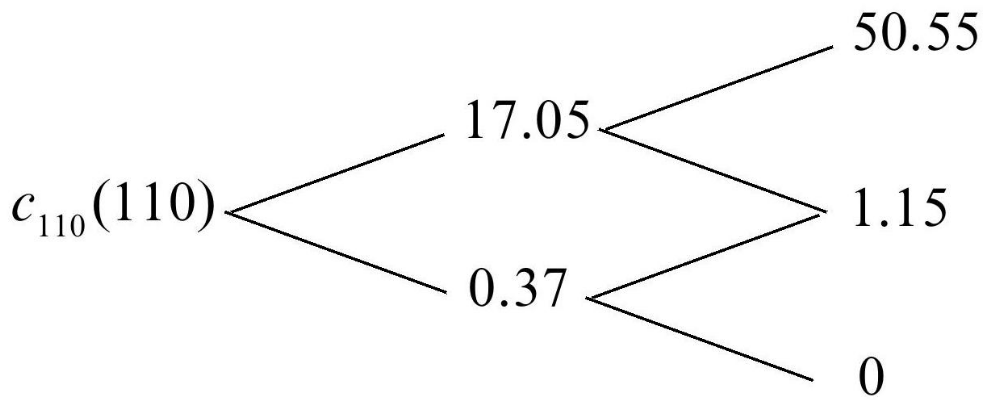
- Work backward through the tree, deciding at each node whether to exercise or wait.
- Important: We use
  - The pre-dividend price to determine the exercise value of a call.
  - The post-dividend price to determine the exercise value of a put.

Let's work through an example. Consider an American call with an exercise price of 110.

- When ABC distributes a 5% dividend at the end of period one, when the tree looks like:

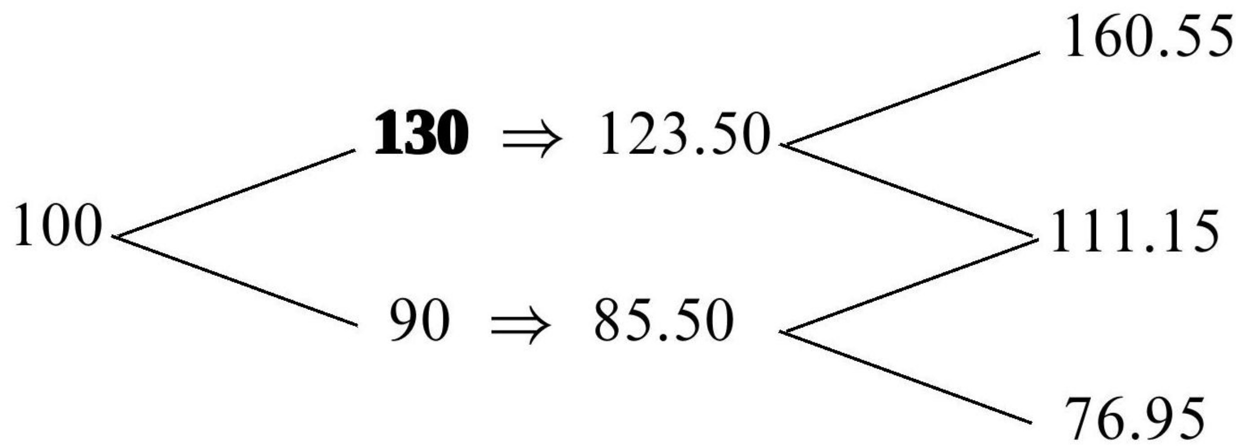


- Here's the payoff diagram for the European call:



- When would you want to exercise early?
- When is intrinsic value > continuation value?
- When the European call's "option value" is worth less than the exercise:

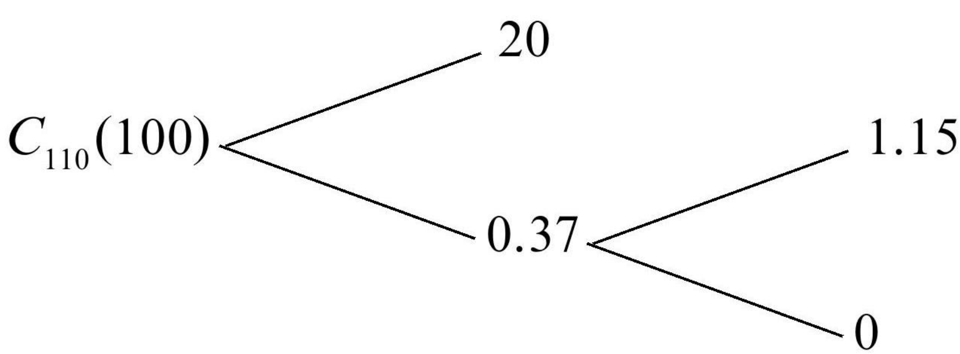
When the stock goes up but before the dividend is paid.



- The value of the European call is then \$17.05...
- ... but if you exercise you get \$130 − \$110 = \$20.

So what's the tree for the American call look like? Just replace the up-node with the higher value.

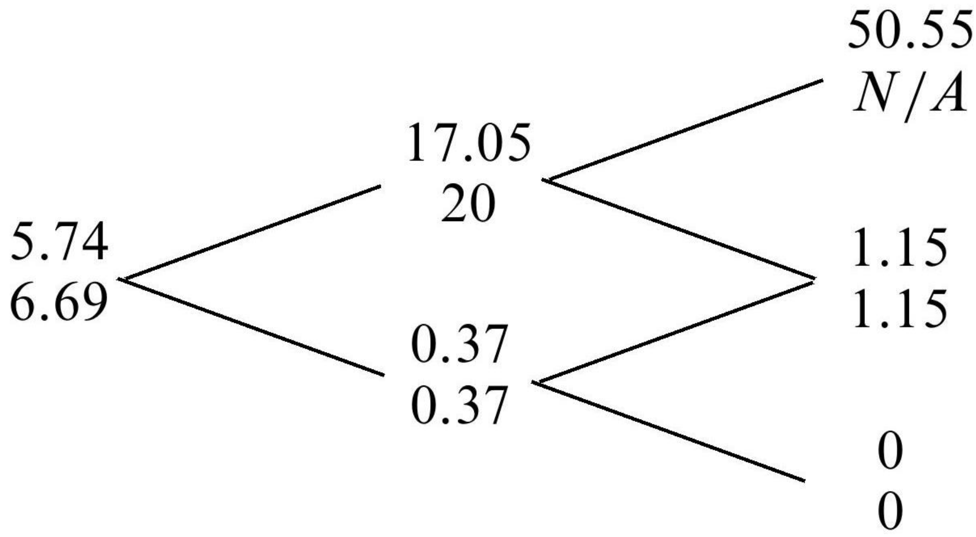
- Payoffs for the American call



- The day-zero call price is then

$$C_{110}(100) = \frac{.33 \times 20 + .67 \times 0.37}{1.033} = 6.69.$$

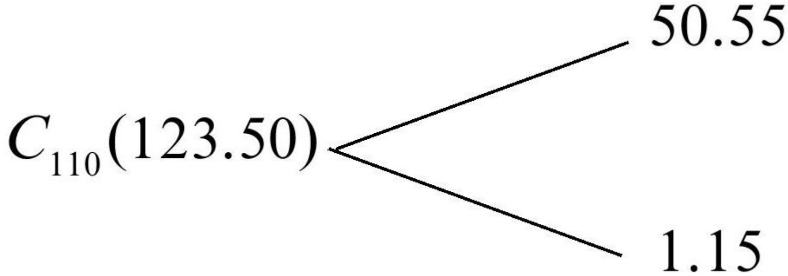
- Let's compare the payoff diagrams for the European call and the American call.



- The difference, $6.69 - 5.74 = \mathbf{0.95}$, is the value of the early exercise option.
  - I.e., it's the value inherent in the additional flexibility provided by the American option.
- At what exercise price would you be (ex ante) indifferent between early and late exercise?
  - Is there one?
  - If so, is high or low?

Dividends and American Options

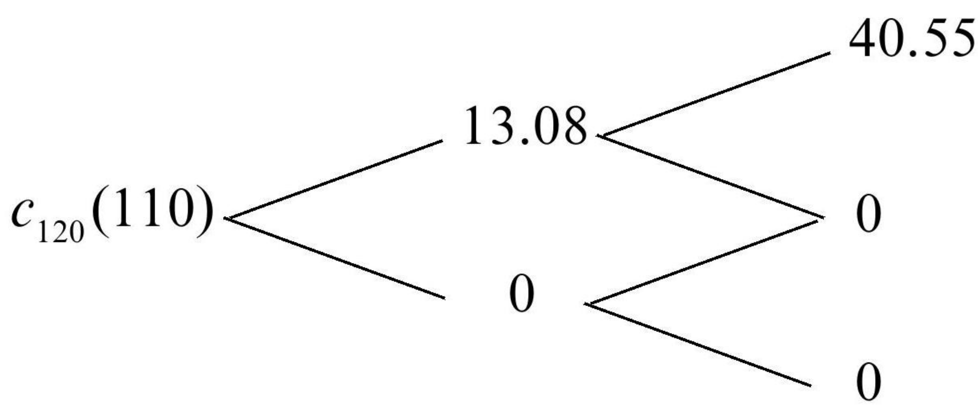
- Let's consider the one-period American call with an exercise price of 110, at the "up-node" and ex dividend:



- This option is worth \$17.05, because you won't exercise early.
  - It's exercise value is only $123.50 - 110 = 13.50$.
- The difference, $17.05 - 13.50 = 3.55$, is the

- Interest on the exercise price: $\left(1 - \frac{1}{1.033}\right) 110 = 3.55$
- The right not to exercise: $3.55 - 3.55 = 0$
- In this example the option was guaranteed to finish in the money (it's a forward, effectively)
- Right not to exercise was worthless.

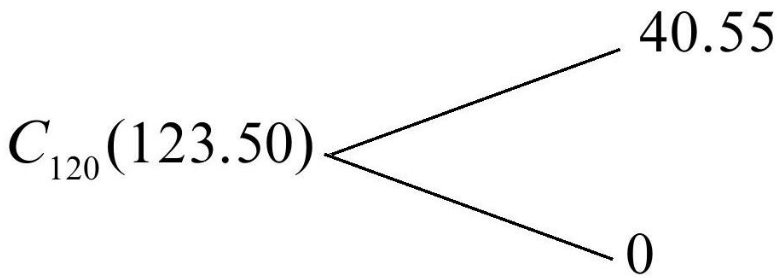
- Exercise cum dividend because $D = 6.5 > 3.55$.
  - I.e., the dividend you capture exceeds the interest you lose and the value of the right not to exercise.
- Then the tree for the call, if you won't exercise, looks like



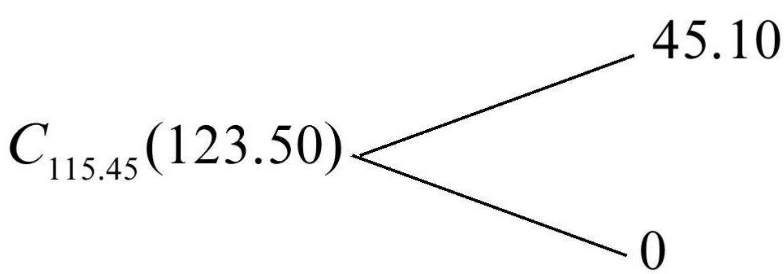
- Would you still want to exercise early?
  - No! Exercise to capture the dividend, and you receive

$$130 - 120 = 10.$$

  - And $10 < 13.08$.
- The higher exercise price makes it less likely you'll exercise early.
  - It increases the interest you'll give up (a little).
  - It increases the value of the right not to exercise (potentially a lot).
- That is, you're more likely to exercise when the option is deeply in the mooney.
- Let's consider the one-period American call with an exercise price of 120 at the "up-node", ex dividend:
  - That's what you receive for not exercising.
  - What do you receive for giving up the dividend?



- This option is worth $\dfrac{40.55/3}{1.033} = 13.08$.
  - Its exercise value is only $123.50 - 120 = 3.50$.
- The difference, $13.08 - 3.50 = 9.58$, is
  - Interest on the exercise price: $\left(1 - \dfrac{1}{1.033}\right) 120 = 3.87$
  - The right not to exercise: $9.58 - 3.87 = 5.71$
  - In this example, the right not to exercise is quite valuable.
- You don't exercise early, because $9.58 > 6.5$.
  - I.e., the dividend you'd capture is less than the interest you'd lose and the value of the right not to exercise.
- We are indifferent if the call has an exercise price of 115.45 :



- This option is worth $\dfrac{45.10/3}{1.033} = 14.55$.
  - Exercise value is only $123.50 - 115.45 = 8.05$.
- The difference is $14.55 - 8.05 = 6.50$
  - Exactly the value of the dividend
  - You can still decompose it:
  - Interest on exercise price: $\left(1 - \dfrac{1}{1.033}\right) 115.45 = 3.69$
  - Right not to exercise: $6.50 - 3.69 = 2.81$