More Wiener Processes Practice Problems

Problem 1.
A company's cash position, measured in millions of dollars, follows a generalized Wiener process with a drift rate of 0.5 per quarter and a variance rate of 4.0 per quarter. How high does the company's initial cash position have to be for the company to have a less than 5% chance of a negative cash position by the end of one year?

Problem 2.
Consider a random variable, Y, that follows the process

$$dY = \mu \, dt + \sigma \, dz$$

For the first three years, $\mu = 2$ and $\sigma = 3$; for the next three years, $\mu = 3$ and $\sigma = 4$. If the initial value of the variable is 5, what is the probability distribution (i.e., mean and variance) of the value of the variable at the end of year six? At the end of year six, what is the probability that $Y < 0$?

Problem 3.
A stock price is currently $80. Its expected return and volatility are 8% and 25%, respectively. What is the probability that the stock price will be greater than $100 in two years? (Hint $S_T > 100$ when $\ln S_T > \ln 100$.)