Wiener Processes
(Finance 4366 Class Problems)

1. A variable, x, starts at 10 and follows a generalized Wiener process

$$dx = adt + bdz$$

where $a = 2$, $b = 3$, and dz is a Wiener process.

(i) What is the mean value of the variable after three years?

SOLUTION: $E(x + dx) = 10 + adt = 10 + 2(3) = 16$.

(ii) What is the standard deviation of the value of the variable after three years?

SOLUTION: $\sigma_x = b\sqrt{dt} = 3\sqrt{3} = 5.19$.

(iii) What is the mean value of the variable after six months?

SOLUTION: $E(x + dx) = 10 + adt = 10 + 2(.5) = 11$.

(iv) What is the standard deviation of the value of the variable after six months?

SOLUTION: $\sigma_x = b\sqrt{dt} = 3\sqrt{.5} = 2.12$.

2. A variable, x, starts at 10 and follows a generalized Wiener process

$$dx = adt + bdz$$

During the first two years $a = 4$ and $b = 3$. During the following three years $a = 6$ and $b = 4$.

(i) What is the mean value of the variable at the end of the five years?

SOLUTION: $E(x + dx) = 10 + a_1dt_1 + a_2dt_2 = 10 + 4(2) + 6(3) = 36$.

(ii) What is the standard deviation of the variable at the end of the five years?

SOLUTION: $\sigma_x = \sqrt{b_1^2dt_1 + b_2^2dt_2} = \sqrt{3^2(2) + 4^2(3)} = 8.12$.