Numerical Example: Pricing Credit Risk

Finance 4366 Class Problem Solutions

Problem Setup:

- Suppose two banks exist which are identical in all respects except for degree of financial leverage.
- At date $t = 0$, Bank 1 issues zero coupon deposits with a face value of $500,000, whereas bank 2 has issued zero coupon deposits with a face value of $800,000.
- Current ($t = 0$) asset value for both banks is $1,000,000, and 1 year from today (at date $t = 1$), depositors expect these banks to pay back the face value of deposits with profits earned from their investments.
- Since both banks are limited liability corporations which hold risky assets, depositors face the risk of default. The continuously compounded rates of return on bank assets are normally distributed, with mean $\mu - .5\sigma^2 = .15 - .5(.16) = 7\%$, and standard deviation $\sigma = .40$.
- The riskless rate of interest is 3%.

1. Suppose there is no deposit insurance. What are the fair market values for the deposits held by Bank 1 and Bank 2 if there is no deposit insurance?

   Solution: In the absence of deposit insurance, depositors are at risk if default occurs; i.e., if $F < B$ at t =1. Consequently, the fair market value of risky deposits is equal to the fair market value of safe deposits minus the value of the limited liability put option; i.e., $V(D) = Be^{-rT} - V(Max[0, B - F])$, where B corresponds to the promised payment and F corresponds to the $t = 1$ value of bank assets.

   We begin by calculating the fair market value for Bank 1 deposits. The value of the limited liability put is $V(Max[0, B - F]) = Be^{-rT}N(-d_2) - V(F)N(-d_1)$, where $d_1 = \dfrac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \dfrac{\ln(1,000,000/500,000) + (.03 + .5(.4^2))1}{.4\sqrt{1}} = 2.0079$ and $d_2 = d_1 - \sigma\sqrt{T} = 2.0079 - .4\sqrt{1} = 1.6079$. Since $N(-d_1) = N(-2.0079) = .0223$ and $N(-d_2) = N(-1.6079) = .0539$, the value of Bank 1's limited liability put option is $V(Max[0, B-F])$ $= 500,000e^{-.03}(.0539) - 1,000,000(.0223) = \$3,840.40$, and the fair market value of Bank 1's deposits is $V(D) = Be^{-rT} - V(Max[0, B - F]) = 500,000e^{-.03} - \$3,840.40 = \$481,382.37$.

   Next, consider Bank 2. Bank 2's $d_1 = \dfrac{\ln(1,000,000/800,000) + (.03 + .5(.4^2))1}{.4\sqrt{1}} = .8329$ and $d_2 = d_1 - \sigma\sqrt{T} = .8329 - .4\sqrt{1} = .4329$. Since $N(-d_1) = N(-.8329) = .2025$ and $N(-d_2) = N(-.4329) = .3326$, the value of Bank 2's limited liability put option is $V(Max[0, B - F])$ $=800,000e^{-.03}(.3326) - 1,000,000(.2025) = \$55,721.88$, and the fair market value of Bank 2's deposits is $V(D) = Be^{-rT} - V(Max[0, B-F]) = 800,000e^{-.03} - \$55,721.88 = \$720,634.55$.

2. What are the values of Bank 1 and Bank 2 limited liability put options?

   Solution: As calculated in the answer to question (1), the values of the limited liability put options held by Bank 1 and Bank 2 are \$3,840.40 and \$55,721.88 respectively.

3. What are the probabilities of default for Bank 1 and Bank 2?

   Solution: As shown in the solution for question (1), the (risk neutral) probabilities of default are $N(-d_2) = N(-1.6079) = .0539$ for Bank 1 and $N(-d_2) = N(-.4329) = .3326$ for Bank 2. Given how highly leveraged Bank 2 is compared with Bank 1, such a difference in risk neutral default probabilities is to be expected.

   Given that bank asset returns are normally distributed with a 7% mean and 40% standard deviation, these facts imply that the probability that Bank 1 will default on its deposit obligations is $N(z_1)$, where

   $$N(z_1) = N(\ln(500,000/1,000,000) - (.15 - .5(.16))/.4) = N(-1.9078) = 2.82\%.$$

   For Bank 2, the probability of default is equal to $N(z_2)$, where

   $$N(z_2) = (\ln(800,000/1,000,000) - (.15 - .5(.16)))/.4 = N(-.7329) = 23.18\%.$$

4. Calculate yields to maturity and credit risk premiums for Bank 1 and Bank 2.

   Solution: Since $B = V(D)e^{YTM(T)}$, it follows that $YTM = \frac{\ln(B/V(D)}{T} = \frac{\ln(500,000/481,328.37)}{1} = 3.79\%$ for Bank 1, and for Bank 2, $YTM = \frac{\ln(800,000/720,634.55)}{1} = 10.45\%$. The credit risk premium for Bank 1 is $YTM_1 - r = 3.79\% - 3\% = .79\%$ and it is $YTM_2 - r = 10.45\% - 3\% = 7.45\%$ for Bank 2.

5. Suppose the government institutes a compulsory risk-based deposit insurance scheme in which bank deposits are fully insured against the risk of default. What are the fair premiums for deposit insurance paid by Bank 1 and Bank 2?

   Solution: Note that the values of the shortfalls for Banks 1 and 2 are represented by the values of these banks' limited liability put options. Since Bank 1 creates a smaller risk of default due to its more conservative financing policy compared with Bank 2, the fair premium for Bank 1 is \$3,840.40 and the fair premium for Bank 2 is \$55,721.88.

6. What effect will the introduction of compulsory deposit insurance have on the yields to maturity and credit risk premiums that depositors expect from Bank 1 and Bank 2?

   Solution: Since the presence of compulsory deposit insurance ensures that depositors for both banks no longer have to bear any credit risk, credit premiums go to zero in both cases and yields fall to the riskless rate of interest which is $r = 3\%$.

7. Now suppose the government charges premiums based on the average of the fair premiums that Bank 1 and Bank 2 should pay. Analyze the behavioral effects of such a pricing scheme. Specifically, who wins and who loses, and what incentives are conveyed by such a scheme?

   Solution: Since fair premiums are \$3,840.40 for Bank 1 and \$55,721.88 for Bank 2, the average premium is (\$3,840.40 + \$55,721.88)/2 = \$29,781.14. Thus, Bank 1 is required to pay \$29,781.14 - \$3,840.40 = \$25,940.74 more than it is worth, whereas Bank 2 pays too little; i.e., only \$29,781.14 rather than \$55,721.88. This new pricing scheme effectively compels Bank 1's owners to subsidize Bank 2's risk taking. If Bank 1 exits the risk pool (so that it no longer has to overpay for deposit insurance), then the government would need to increase Bank 2's premium. However, it Bank 1 remains in the risk pool, this mispricing will incentivize Bank 1 to become a riskier bank.