Midterm Exam #2 Formula Sheet

Put-call parity theorem (assuming no dividends; options are European)

where
$\delta t =$ the length of a timestep; e.g., if the timestep is one year, then $\delta t = 1$.
$n =$ the number of timesteps until expiration;
$c =$ value of a call option with an exercise price of K and n timesteps until expiration;
$p =$ value of a put option with an exercise price of K and n timesteps until expiration;
$r =$ the annualized riskless rate of interest; and
$S =$ the value of the underlying asset.

Delta Hedging Approach for Option Pricing (1 timestep call and put)

- Initial hedge portfolio value for call option: $V_H = c - \Delta S$
- u node hedge portfolio value for call option: $V_H^u = c_u - \Delta S_u$
- d node hedge portfolio value for call option: $V_H^d = c_d - \Delta S_d$
- Set $V_H^u = V_H^d \to \Delta = \dfrac{c_u - c_d}{S_u - S_d}$. Then $V_H = c - \Delta S = e^{-r\delta t} V_H^u = e^{-r\delta t} V_H^d$ and either of these equations can solved for the arbitrage-free price of the call, c.
- Put solution procedure is similar to the call solution procedure, except hedge portfolio values are $V_H = p + \Delta S$, $V_H^u = p_u + \Delta S_u$, and $V_H^d = p_d + \Delta S_d$.

Replicating Portfolio Approach for Option Pricing (for a 1 timestep call or put)

- u node replicating portfolio value for option (call or put): $V_{RP}^u = \Delta S_u + e^{r\delta t} B$
- d node replicating portfolio value for option (call or put): $V_{RP}^d = \Delta S_d + e^{r\delta t} B$
- At the tree's inception, $f = V_{RP} = \Delta S + B$, where f is the arbitrage-free price of the option, $\Delta = \dfrac{f_u - f_d}{S_u - S_d}$, $B = \dfrac{u f_d - d f_u}{e^{r\delta t}(u - d)}$, and f_u (f_d) correspond to option value at node u (d).

Risk Neutral Valuation Formula (for a 1 timestep call or put):

$$f = e^{-r\delta t}[q f_u + (1 - q) f_d], \text{ where } q = \frac{e^{r\delta t} - d}{u - d}.$$

Risk Neutral Valuation Formula for an n timestep European call option (AKA the "Cox-Ross-Rubinstein" model):

$$c = e^{-rn\delta t} \left[\sum_{j=a}^{n} \left(\frac{n!}{j!\,(n-j)!} \right) q^j (1-q)^{n-j} \left(u^j d^{n-j} S - K \right) \right],$$

where $a =$ the smallest integer value $> \dfrac{\ln(K/Sd^n)}{\ln(u/d)}$.

Black-Scholes-Merton Call and Put Option Pricing Formulas

$$c = SN(d_1) - e^{-r(T-t)}KN(d_2) \text{ and } p = e^{-r(T-t)}KN(-d_2) - SN(-d_1),$$

where

$d_1 = \dfrac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}}$;

$d_2 = d_1 - \sigma\sqrt{T}$;

σ = annualized volatility of underlying asset's rate of return;

$N(d_1)$= standard normal distribution evaluated at d_1;

$N(d_2)$= standard normal distribution evaluated at d_2;

$N(-d_1)$ = standard normal distribution evaluated at $-d_1$; and

$N(-d_2)$= standard normal distribution evaluated at $-d_2$.

Basic Wiener Process

$$dz = \varepsilon\sqrt{dt},$$

where ε is a standard normal random variable, $E(\varepsilon) = 0$, $\sigma^2_\varepsilon = 1$, $E(dz) = 0$, and $\sigma^2_{dz} = dt$.

Generalized Wiener Process

$$dx = adt + bdz,$$

where $E(dx) = adt$ and $\sigma^2_{dx} = b^2dt$,

Geometric Brownian Motion

$$dS = \mu Sdt + \sigma Sdz$$

where:

- S is the stock price at time t;
- μ is the annualized expected return on the underlying asset;
- σ is annualized volatility or standard deviation of the return on the underlying asset;
- dz is the basic Wiener process; and
- dt is the time increment.