Properties of Stock Options

Factors Affecting Option Prices

Variable	**Call Option**	**Put Option**
Current Stock Price	+	-
Exercise Price	-	+
Time to Expiration	+	?
Volatility	+	+
Risk-Free Rate	+	-
Dividends	-	+

Figure 10.1 Effect of changes in stock price, strike price, and expiration date on option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.

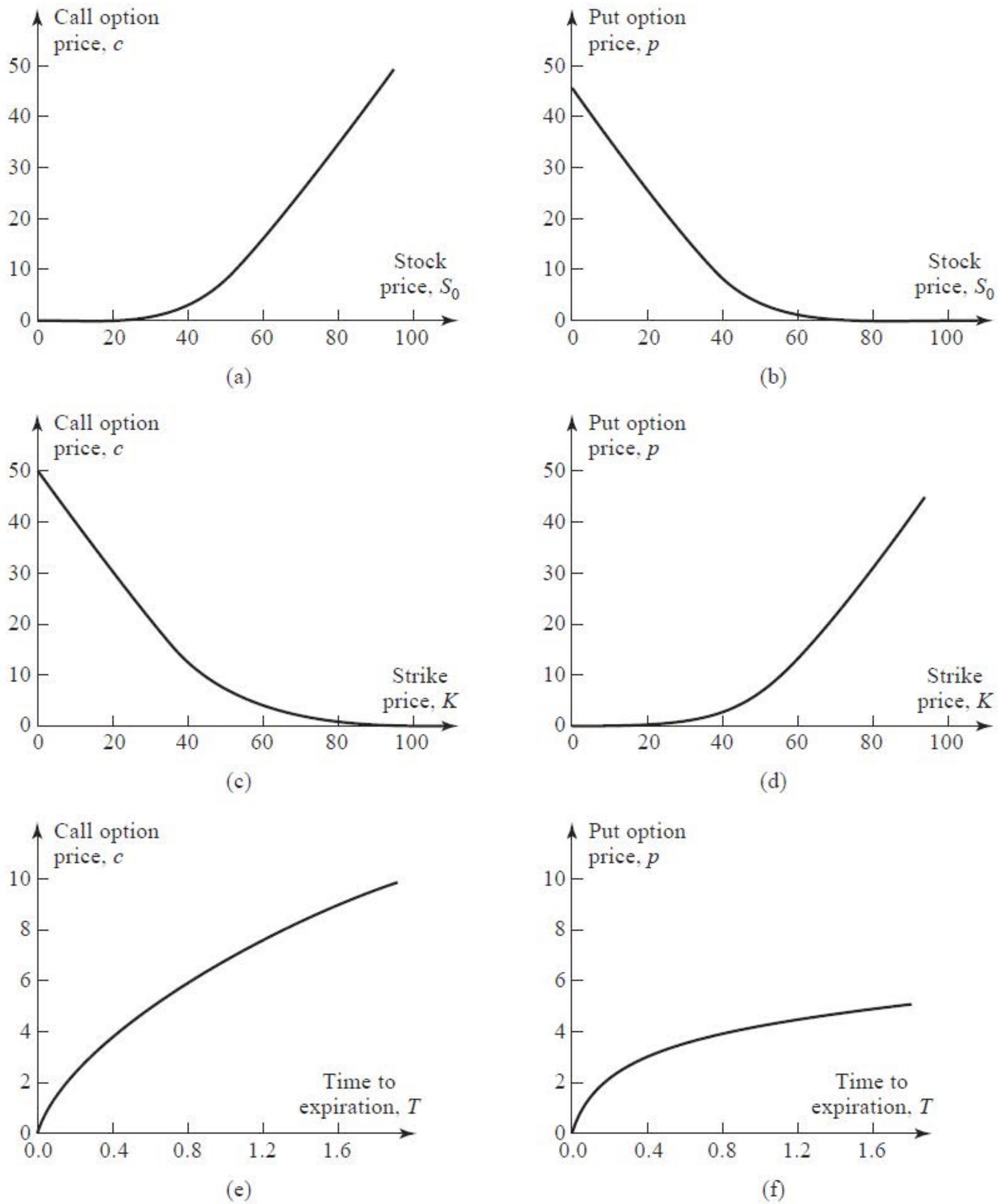
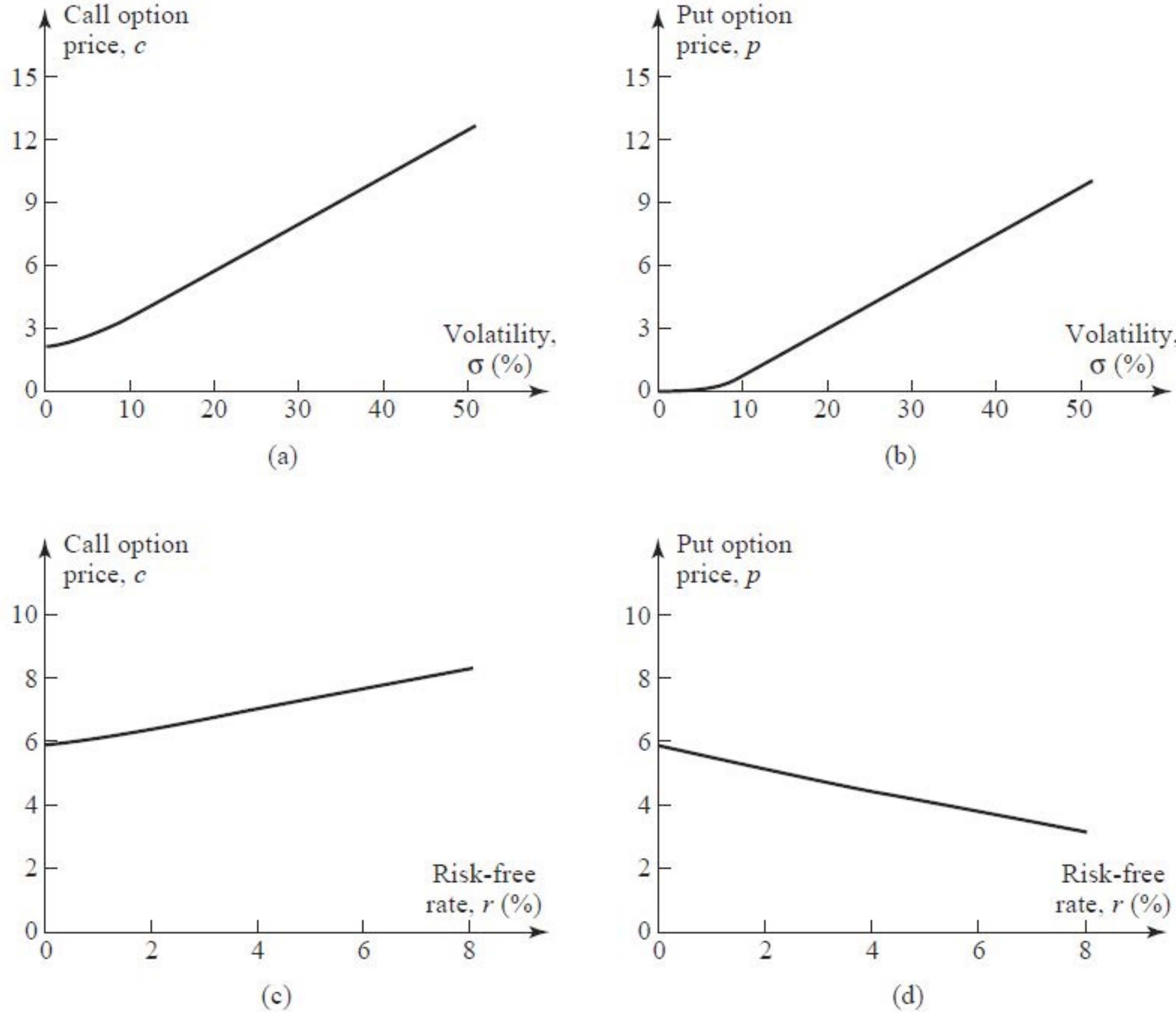


Figure 10.2 Effect of changes in volatility and risk-free interest rate on option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.



Assumptions and Notation

- c: European call option price
- p: European put option price
- $S(t)$: Stock price at date t
- K: Strike or exercise price
- $T - t$: Remaining life of option
- σ: Volatility of stock price
- C: American Call option price
- P: American Put option price
- D: Present value of dividends during the option's life
- r: Risk-free rate for maturity T with continuous compounding

No-Arbitrage Bounds on Options

- A call option is never worth more than the underlying stock:

$$C(S,K,t,T) \le S(t) \ \ \& \ c(S,K,t,T) \le S(t)$$

- A put option is never worth more than the exercise price:

$$P\,(S,K,t,T) \le K \ \ \& \ \ p(S,K,t,T) \le K$$

No-Arbitrage Bounds on Options

- A European put option is never worth more than the present value of the strike price:

$$p(S, K, t, T) \leq Ke^{-r(T-t)} < K$$

- This is because the payoff at maturity of a European put option cannot exceed K.

No-Arbitrage Bounds on Options

- Options never have negative value:

$$c(S, K, t, T) \geq 0 \ \ \& \ \ C(S, K, t, T) \geq 0$$

$$p(S, K, t, T) \geq 0 \ \ \& \ \ P(S, K, t, T) \geq 0$$

- American options are at least as valuable as European options:

$$C(S, K, t, T) \geq c(S, K, t, T)$$

$$P(S, K, t, T) \geq p(S, K, t, T)$$

No-Arbitrage Bounds on Options

- American options with more time to maturity are at least as valuable as the same options with less time to maturity:

$$C(S, K, t, T_2 > T_1) \geq C(S, K, t, T_1)$$

$$P(S, K, t, T_2 > T_1) \geq P(S, K, t, T_1)$$

- European call options with more time to maturity are at least as valuable as the same options with less time to maturity:

$$c(S, K, t, T_2 > T_1) \geq c(S, K, t, T_1)$$

No-Arbitrage Bounds on Options

- An American option is worth at least its exercised value (the payoff you receive if you exercise today):

$$C(S, K, t, T) \geq \max[0, S(t) - K]$$

$$P(S, K, t, T) \geq \max[0, K - S(t)]$$

- Note: no such restriction exists for European options because exercise may only occur at date T.

No-Arbitrage Bounds on Options

- The price of a call option satisfies:

$$c(S, K, t, T) \geq \max[0, S(t) - Ke^{-r(T-t)}]$$

$$C(S, K, t, T) \geq \max[0, S(t) - Ke^{-r(T-t)}]$$

Proof: We only need to show that:

$$c(S, K, t, T) \geq \max[0, S(t) - Ke^{-r(T-t)}],$$

since $C(S, K, t, T) \geq c(S, K, t, T)$ (see p. 8)

Calls: An Arbitrage Opportunity?

- Suppose that

$c = 3$ $\qquad$ $S_0 = 20$

$T = 1$ $\qquad$ $r = 10\%$

$K = 18$ $\qquad$ $D = 0$

- Is there an arbitrage opportunity?

No-Arbitrage Bounds on Options

- The price of a put option satisfies:

$$p(S, K, t, T) \geq \max[0, Ke^{-r(T-t)} - S(t)]$$

$$P(S, K, t, T) \geq \max[0, Ke^{-r(T-t)} - S(t)]$$

<u>Proof:</u> We only need to show that:

$$p(S, K, t, T) \geq \max[0, Ke^{-r(T-t)} - S(t)],$$

since $P(S, K, t, T) \geq p(S, K, t, T)$. (see p. 8)

Puts: An Arbitrage Opportunity?

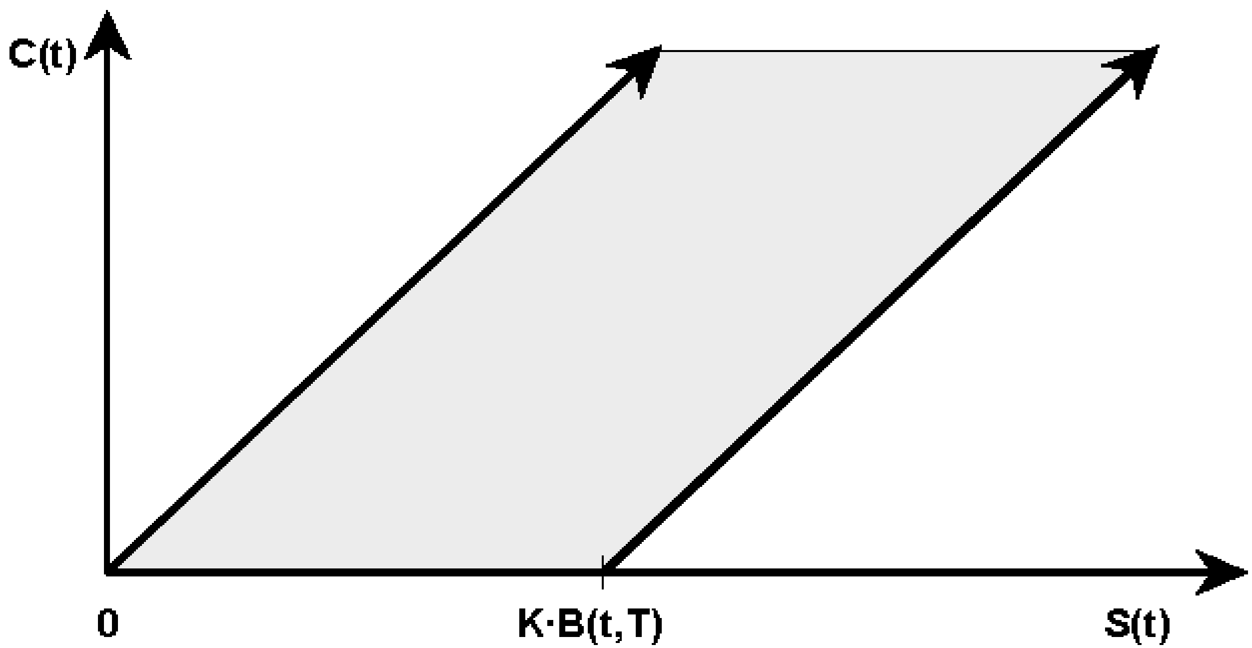
- Suppose that

$$p = 1 \qquad S_0 = 37$$
$$T = 0.5 \qquad r = 5\%$$
$$K = 40 \qquad D = 0$$

- Is there an arbitrage opportunity?

No-Arbitrage Bounds on Options

Since $\max[0, S(t) - Ke^{-r(T-t)}] \leq c \leq S(t)$, this implies that the value of a European call option on a non dividend paying stock lies within the shaded region shown below:



No-Arbitrage Bounds on Options

Since $\max[0, Ke^{-r(T-t)} - S(t)] \leq p \leq Ke^{-r(T-t)}$, this implies that the value of a European put option on a non dividend paying stock lies within the shaded region shown below: