Problem #1 (32 points)

You are given the following set of prices:

Security	Maturity (years from today)	Exercise Price	Current Price
TC stock	-	-	$94
European Put Option on TC stock	1	$80	$5
European Call Option on TC	1	$80	?
Zero Coupon Treasury Bill (par/maturity value = $100 from today)	1	-	$91

A. (16 points) What is the price of the call option (assuming that TC stock does not pay dividends)?

SOLUTION: Using put-call parity, we find that $c = p + S - e^{-rT}K = 5 + 94 - 80^{*}.91 =$ $26.20.

B. (16 points) Now suppose that the market price of a TC call option (with an exercise price of $80) is $30. When you tell your boss that this call option is too expensive, he tells you that you must be wrong; after all, the formula which you used to price the call option is "just" a theory and perhaps the market knows something that you don't concerning the future (possibly "rosy") prospects for TC stock. Prove that your boss is wrong by applying the "arbitrage-free" argument.

SOLUTION: The put-call parity equation relies on the no-arbitrage argument, and if it is violated you can make risk-free profit in the market. Since the call is overpriced, you write (sell) a call and buy a replicating portfolio, which involves buying one unit of stock, buying a put with an exercise price of $80 and borrowing $PV(\$80) = \$72.80$. This strategy will cost you $26.20, but you get $30 from writing the call. Now you have a net cash flow of $3.80, and you have a net cash flow of zero in the future.

Problem #2 (32 points)

Suppose the riskless rate of interest is 0%, the price of a riskless zero-coupon bond is $1, and the price of a (non-dividend paying) stock is $1. In the future, only two equally probable outcomes exist for the economy, good and bad. In the good state, the stock is worth $2, whereas in the bad state, the stock is worth $.50.

A. (8 points) Show that the replicating portfolio for a European call option with an exercise price of $1 comprises a long position in 2/3 of a share of stock and a short position in 1/3 of one bond (Hint: you can either show this analytically or numerically by demonstrating that the state-contingent payoffs on the replicating portfolio and the call option are identical).

SOLUTION: If I hold a portfolio comprising 2/3 of a share of stock and a short position in 1/3 of one bond, then in the good state, this portfolio pays off $(2/3)(2) - (1/3)(1) = \$1$, and in the bad state, this portfolio pays off $(2/3)(.5) - (1/3)(1) = \$0$. This replicates the payoff on the call option; since $c(T) = \max(0, S(T) - K)$, in the good state the call pays off $\max(0, 2-1) = \$1$, and in the bad state the call pays off $\max(0, .5-1) = \$0$.

B. (8 points) What is the "arbitrage-free" price for this call option?

SOLUTION: Since the replicating portfolio (which comprises 2/3 of a share of stock and a short position in 1/3 of one bond) is worth $(2/3)(1) - (1/3)(1) = \$0.33$, then the call option must be worth this same amount; otherwise riskless arbitrage profits could be earned.

C. (8 points) Show that the replicating portfolio for a European put option with an exercise price of \$1 comprises a short position in 1/3 of a share of stock and a long position in 2/3 of one bond (Hint: you can either show this analytically or numerically by demonstrating that the state-contingent payoffs on the replicating portfolio and the put option are identical).

SOLUTION: If I hold a portfolio comprising $-1/3$ of a share of stock and 2/3 of one bond, then in the good state, this portfolio pays off $(2/3)(1) - (1/3)(2) = \$0$, and in the bad state, this portfolio pays off $(2/3)(1) - (1/3)(.5) = \$.5$. This replicates the payoff on the put option; since $p(T) = \max(0, K - S(T))$, in the good state the put pays off $\max(0, 1-2) = \$0$, and in the bad state the put pays off $\max(0, 1 - .5) = \$.50$.

D. (8 points) What is the "arbitrage-free" price of a put option on the stock with an exercise price of \$1 ?

SOLUTION: Since the replicating portfolio (which comprises $-1/3$ of a share of stock and a long position in 2/3 of one bond) is worth $-(1/3)(1) + (2/3)(1) = \$0.33$, then the put option must be worth this same amount; otherwise riskless arbitrage profits could be earned. Note also that one can simply invoke the put-call parity equation to establish the price for the put; since the call is worth \$0.33 and the stock and the bond are both worth \$1 each, it follows that $p = c + e^{-rT}K - S = \$0.33 + \$1 - \$1 = \$0.33$.

Problem #3 (32 points)

Suppose the euro is currently trading at \$1.50 and that one-year zero-coupon bonds (with face values of 100) in the U.S. and Europe are trading at \$95 and \$93.10 respectively.

A. (8 points) What is the one-year forward price of the euro?

SOLUTION: We can infer the one-year U.S. and European interest rates from the prices of the U.S. and the euro zero-coupon bonds; since U.S. bonds are worth $\$95 = e^{-r_\$}\$100$, this implies that $r_\$ = \ln(100/95) = 5.13\%$, and since euro bonds are worth $\$93.10 = e^{-r_\epsilon}\$100$, this implies that $r_\epsilon = \ln(100/93.10) = 7.15\%$. Thus, the one-year forward price of the euro is

$$F(t,T) = S(t) \times e^{(r_\$ - r_\epsilon)\times(T-t)} = \$1.5e^{(.0513-0715)} = \$1.47$$

B. (8 points) What is the replicating portfolio for a one-year euro forward contract (based upon the forward price that you calculated in part (A))?

SOLUTION: The forward contract delivers one euro one year from today. The replicating portfolio comprises a long euro bond position currently worth $S(t) \times e^{-r_\epsilon \times (T-t)} = \$1.5e^{-.0715} = \$1.3965$ and a short U.S. bond position currently worth $F(t,T) \times e^{-r_\$ \times (T-t)} = \$1.47e^{-.0513} = \$1.3965$; thus today's value of the replicating portfolio $S(t) \times e^{-r_\epsilon \times (T-t)} - F(t,T) \times e^{-r_\$ \times (T-t)} = \$1.3965 - \$1.3965 = \$0$, which of course corresponds to the initial value of the forward contract.

C. (8 points) Suppose the price of a one-year Euro forward contract is \$1.49. Outline a trading strategy that would enable you to you make a riskless arbitrage profit. How much profit would you earn from implementing such a strategy?

SOLUTION: At \$1.49, the one-year Euro forward contract is too expensive, so we would want to sell Euros forward, borrow Euros and lend dollars. The following table outlines the riskless arbitrage strategy:

Transaction	now	One year from now
Sell Forward	0	$\$1.49 - S_T$
Lend $e^{-r_\epsilon} = e^{-.0715} = .931$ euros @ \$1.50	$-\$1.3965$	S_T
Borrow $e^{-r_\$} = e^{-.0513} = .95$ dollars @ \$1.47	\$1.3965	$-\$1.3965e^{.0513} = -\$1.47$
Total	0	\$0.02

D. (8 points) Suppose that you "buy" the forward contract for the price that you calculated in part (A), and immediately after your purchase, the Fed raises interest rates. With the rate hike,

- The U.S. bond falls in value from \$95 to \$93.10,
- The Euro bond price remains unchanged, and
- The dollar strengthens, with the dollar value of the Euro falling to \$1.48.

If you immediately close out the contract (i.e., sell it), how much money will you make or lose?

SOLUTION: Since this change in Fed policy occurs immediately, the value of the replicating portfolio is now $e^{-.0715}(S(t) - F(t,T)) = .931(1.48 - 1.47) = \$0.00931$. Thus the profit that I earn from the scenario described above is \$0.00931 per euro that I bought forward.