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Options, Futures and Other Derivatives — Name: SOLUTIONS
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Problem Set 3

Problem 1.

The T&Q index spot price is $1,100 per share, the riskless rate of interest is 5%, and the continuous dividend yield on the index is 2%.

A. What is the arbitrage-free 6-month forward price for a share of the T&Q index?

SOLUTION: First, we need to find the fair value of the forward price. We plug the continuously compounded interest rate, the dividend yield, and the time to expiration in years into the valuation formula and notice that the time to expiration is six months; i.e., $T - t = 0.5$ years. We have:

$$F(t,T) = S(t) \times e^{(r-\delta)\times(T-t)} = \$1,100 \times e^{(0.05-0.02)\times 0.5} = \$1,100 \times 1.01511 = \$1,116.62.$$

B. Suppose the 6-month T&Q index forward contract is trading at $1,120. Outline a trading strategy involving a combination of the forward contract, the index, and a riskless bond, producing riskless profit with zero net investment. How much profit will this trading strategy produce?

SOLUTION: If the market value of the forward price is $1,120 per share, then the forward contract is too expensive relative to the fair per share value we have determined. Therefore, we will sell forward at $1,120, and create a synthetic forward for $1,116.82, making a sure profit of $3.38:

Transaction	Today	In six months
Sell forward	0	\$1,120.00 - S_T
Buy one share of the T&Q index (net of dividend)	-\$1,100 × .99 = -\$1,089.055	S_T
Borrow \$1,089.055	\$1,089.055	-\$1,116.62
TOTAL	0	\$3.38

This arbitrage strategy requires no initial investment, has no risk, and has a strictly positive payoff. Here, we have exploited the overpricing of the forward contract with a pure arbitrage strategy in which we sell forward and buy the replicating portfolio.

C. Now suppose the 6-month T&Q index forward contract is trading at $1,110 per share. Outline a trading strategy involving a combination of the forward contract, the index, and a riskless bond which produces riskless profit with zero net investment. How much profit will this trading strategy produce?

SOLUTION: If the market value of the forward price is \$1,110 per share, then the forward contract is too expensive, relative to the fair per share value we have determined. Therefore, we will buy forward at \$1,110, and create a synthetic short forward for \$1,116.62, thus making a sure profit of \$6.62:

Transaction	Today	In six months
Buy forward	0	S_T - \$1,110.00
Short one share of the T&Q index (net of dividend)	\$1,100 × .99 = \$1,089.055	-S_T
Lend \$1,089.055	-\$1,089.055	\$1,116.62
TOTAL	0	\$6.62

As in part B, this arbitrage strategy requires no initial investment, has no risk, and has a strictly positive payoff. Here, we have exploited the underpricing of the forward contract with a pure arbitrage strategy in which we buy forward and sell the replicating portfolio.

Problem 2.

Suppose the current price for a riskless discount bond that pays \$50 in one year is \$49, and the current price for a (non-dividend paying) stock is \$50. One year from today, only two outcomes exist for the economy, good or bad. In the good state, the stock will be worth \$100, whereas in the bad state, the stock will be worth \$25.

A. Determine the replicating portfolio for a European call option with an exercise price of \$40.

SOLUTION: Note that the value of the call option is equal to the value of the replicating portfolio for the call option in both the future good and bad states; i.e.,

$$c_{1,g} = \max[0, 100 - 40] = \$60 = V_{1,g} = \Delta 100 + \beta(50); \text{and}$$
$$c_{1,b} = \max[0, 25 - 40] = \$0 = V_{1,b} = \Delta 25 + \beta(50).$$

Since we have two equations in two unknowns, we can solve for Δ and β by subtracting the equation for $V_{1,b}$ from the equation for $V_{1,b}$:

$$\Delta 100 + \beta(50) - (\Delta 25 + \beta(50)) = 60;$$
$$\therefore \Delta 75 = 60 \Rightarrow \Delta = 60/75 = .8 \Rightarrow \beta = -.4.$$

Thus, if I hold a portfolio consisting of 4/5 of a share of stock and a short position in 2/5 of one bond, then in the good state, this portfolio pays off (4/5)(100) – (.4)(50) = \$60, and in the bad state, this portfolio pays off (4/5)(25) – (.4)(50) = \$0. Furthermore, these payoffs perfectly replicate the call option payoffs; since $c_{T,g} = \max[0, 100 - 40] = \$60$ in the good state and $c_{T,b} = \max[0, 25 - 40] = \$0$ in the bad state.

B. What is the "arbitrage-free" price for this call option?

SOLUTION: Since the current market value of replicating portfolio for the call option is (4/5)(50)-(2/5)(49) = \$20.40, then the call option *must* be worth this same amount; otherwise riskless arbitrage profits could be earned.

C. Determine the replicating portfolio for a European put option with an exercise price of \$40.

SOLUTION: Note that the value of the put option is equal to the value of the replicating portfolio for the put option in both the future good and bad states; i.e.,

$$p_{1,g} = \max[0, 40 - 100] = \$0 = V_{1,g} = \Delta 100 + \beta(50); \text{and}$$
$$p_{1,b} = \max[0, 40 - 25] = \$15 = V_{1,b} = \Delta 25 + \beta(50).$$

Since we have two equations in two unknowns, we can solve for Δ and β by subtracting the equation for $V_{1,b}$ from the equation for $V_{1,b}$:

$$\Delta 100 + \beta(50) - (\Delta 25 + \beta(50)) = -15;$$
$$\therefore \Delta 75 = -15 \Rightarrow \Delta = -15/75 = -.2 \Rightarrow \beta = .4.$$

Thus, if I hold a portfolio consisting of a short position in 1/5 of a share of stock and a long position in 2/5 of one bond, then in the good state, this portfolio pays off (-1/5)(100) + (2/5)(50) = \$0, and in the bad state, this portfolio pays off (-1/5)(25) + (2/5)(50) = \$15. Furthermore, these payoffs perfectly replicate the put option payoffs; since $p_{T,g} = \max[0, 40 - 100] = 0$ in the good state and $p_{T,b} = \max[0, 40 - 25] = \$15$ in the bad state.

D. What is the "arbitrage-free" price of a put option on the stock with an exercise price of \$40?

SOLUTION: Since the current market value of replicating portfolio for the put option is -(1/5)(50) + (2/5)(49) = \$9.60, then the put option *must* be worth this same amount; otherwise riskless arbitrage profits could be earned. Note also that one can simply invoke the put-call parity equation to establish the price for the put; since the continuously compounded rate of interest is ln(50/49) = 2% and the call option and stock are worth \$20.40 and \$50 respectively, it follows that $p = c + e^{-rT}K$- S = \$20.40 + \$39.20 - \$50 = \$9.60.