Baylor University
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Department of Finance, Insurance & Real Estate

Options, Futures and Other Derivatives
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Problem Set 4

Name: SOLUTIONS

Problem 1 (20 points).

The price of a non-dividend paying stock is \$19 and the price of a six-month European call option on the stock with an exercise price of \$20 is \$1. The riskless rate is 4% per year. What is the price of a six-month European put option with an exercise price of \$20?

SOLUTION: In this case, $c_t = 1$, $T - t = 0.25$, $S_t = 19$, $K = 20$, and $r = 0.04$. From put-call parity,

$$p_t = c_t + Ke^{-r(T-t)} - S_t \rightarrow p_t = 1 + 20e^{-0.04\times 0.5} - 19 = \$1.60.$$

Problem 2 (20 points).

What is the lower "no arbitrage" boundary for the price of a six-month call option on a non-dividend-paying stock when the stock price is \$80, the exercise price is \$75, and the riskless interest rate is 10% per year?

SOLUTION: The lower "no arbitrage" call option price boundary is:

$$c_t = S_t - Ke^{-r(T-t)} = 80 - 75e^{-0.1\times 0.5} = \$8.66.$$

Problem 3 (20 points).

What is the lower "no arbitrage" boundary for the price of a two-month European put option on a non-dividend-paying stock when the stock price is \$58, the exercise price is \$65, and the riskless interest rate is 5% per year?

SOLUTION: The lower "no arbitrage" put option price boundary is:

$$p_t = Ke^{-r(T-t)} - S_t = 65e^{-0.05\times 2/12} - 58 = \$6.46.$$

Problem 4 (20 points).

The price of a European call that expires in six months and has an exercise price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months, and the riskless interest rate is 10% per year. What is the price of a European put option that expires in six months and has an exercise price of \$30?

SOLUTION: The (cum dividend) put-call parity equation is $c_t + Ke^{-r(T-t)} + D = p_t + S_t$, where D represents the present value of dividends paid during the life of the option. Solving for the put price p_t, this implies that $p_t = c_t + Ke^{-r(T-t)} + D - S_t$. Thus,

$$p_t = 2 + 30e^{-0.1\times 6/12} + (0.5e^{-0.1\times 2/12} + 0.5e^{-0.1\times 5/12}) - 29 = 2.51.$$

Problem 5 (20 points).

Historically, exchange-traded European call options existed before exchange-traded European put options were ever introduced. Describe how an investor could have traded a "synthetic" European put option on a non-dividend-paying stock back when calls were traded but puts were not.

SOLUTION: The put-call parity equation $c_t + Ke^{-r(T-t)} = p_t + S_t$ can be rearranged to solve for the price of a synthetic put option; specifically, $p_t = c_t + Ke^{-r(T-t)} - S_t$. In plain English, a synthetic European put can be created by 1) buying an otherwise identical call option, 2) shorting the stock, and 3) keeping an amount of cash that, when invested at the riskless rate, will grow enough to cover the short stock position in case if the call option is in-the-money at the expiration date. If this happens, then the investor exercises the call option, buys the stock for the exercise price, and uses this share of stock to close out the short stock position, yielding a net payoff on the synthetic put equal to $0. However, if the call option is out-of-the-money at the expiration date, the call option is not exercised, and the short position is closed out for a gain equal to the put payoff.