Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

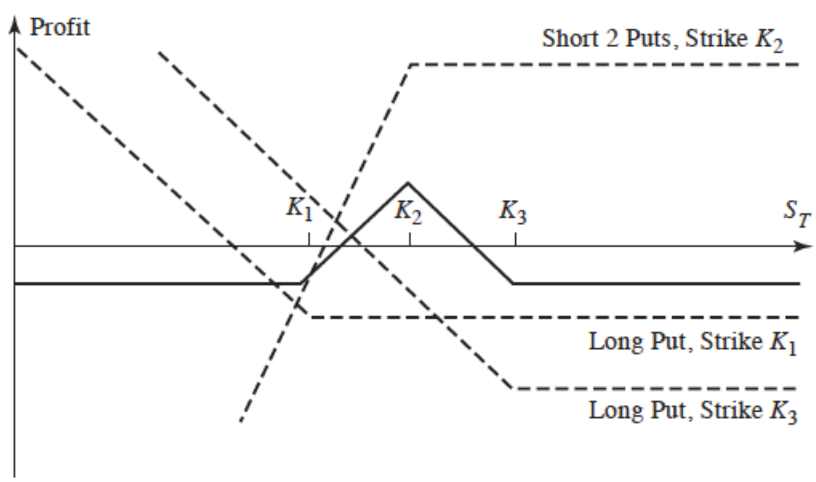
Options, Futures and Other Derivatives — Name: SOLUTIONS
Dr. Garven
Problem Set 5

Problem 1 (40 points).
Three put options on a stock have the same expiration date and strike prices of \$55, \$60, and \$65. The market prices are \$3, \$5, and \$8, respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy. For what range of stock prices would the butterfly spread lead to a loss?

SOLUTION: A butterfly spread is created by buying the \$55 put, buying the \$65 put and selling two of the \$60 puts. This costs $3+8-2\times 5=\$1$ initially. The following table shows the profit/loss from the strategy.

Stock Price	*Payoff*	*Profit*
$S_T \geq 65$	0	-1
$60 \leq S_T < 65$	$65 - S_T$	$64 - S_T$
$55 \leq S_T < 60$	$S_T - 55$	$S_T - 56$
$S_T < 55$	0	-1

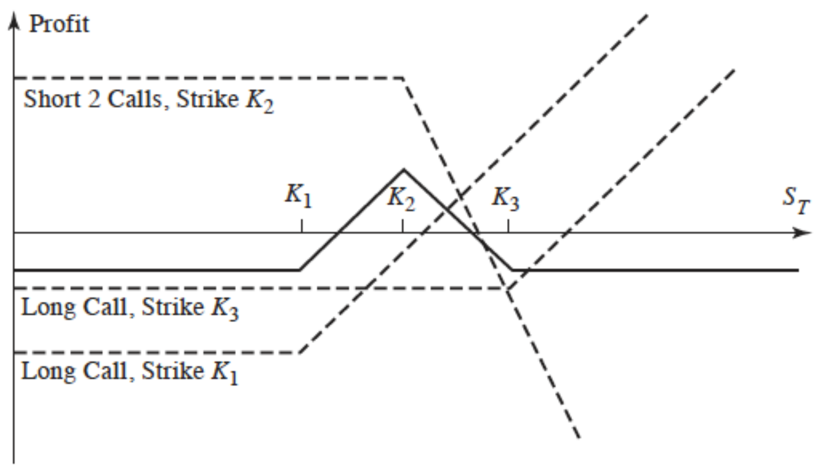
The butterfly spread leads to a loss when the final stock price is greater than \$64 or less than \$56; the source of loss is the initial net cost of \$1 incurred in setting up the butterfly spread strategy, since date T payoffs on the spread itself are non-negative. The (net of setup cost) payoffs shown in the table above resemble the following payoff diagram:



Problem 2 (40 points).
Suppose that c_1, c_2, and c_3 are the prices of European call options with exercise prices K_1, K_2, and K_3, respectively, where $K_3 > K_2 > K_1$ and $K_3 - K_2 = K_2 - K_1$. All options have the same time to expiration. With these options, form a Date T Portfolio that is long one option with exercise price K_1, long one option with exercise price K_3, and short two options with exercise price K_2.

1. Identify and explain what kind of strategy this is (hint: the possible choices include bull, bear, butterfly, or calendar).

   SOLUTION: This is a butterfly spread; the following (net of setup cost) payoff diagram shows how this trading strategy resembles the put-based trading strategy in Problem 1; only here, a similar overall effect obtains from employing call options rather than put options:



2. Show that the price of the call option with exercise price K_2 (i.e., c_2) must be less than or equal to half of the sum of the prices of the call options with the K_1 and K_3 exercise prices (i.e., c_1 and c_3); i.e., show that $c_2 \leq 0.5(c_1 + c_3)$.

   SOLUTION: We start by showing that date T payoffs on this butterfly spread are non-negative over the $S_T \leq K_1, K_1 < S_T \leq K_2, K_2 < S_T \leq K_3$, and $S_T > K_3$ terminal share price intervals:

   $S_T \leq K_1 \Rightarrow$ Date T Payoff $= 0$

   $K_1 < S_T \leq K_2 \Rightarrow$ Date T Payoff $= S_T - K_1$

   $K_2 < S_T \leq K_3 \Rightarrow$ Date T Payoff $= S_T - K_1 - 2(S_T - K_2) = K_2 - K_1 - (S_T - K_2) \geq 0$

   $S_T > K_3 \Rightarrow$ Date T Payoff $= S_T - K_1 - 2(S_T - K_2) + S_T - K_3 = K_2 - K_1 - (K_3 - K_2) = 0$

   The payoffs on this butterfly spread are non-negative; i.e., they are always either positive or zero when the options which comprise this spread expire at date T. Since this is true, it must also be true that the current market value of the spread, $c_1 + c_3 - 2c_2 \geq 0$; otherwise an arbitrage opportunity would exist. Solving for c_2, we find that $c_2 \leq 0.5(c_1 + c_3)$.

Problem 3 (20 points).

Draw a picture of trading profits and losses which obtain from selling a call option with an exercise price of K_2 and buying a put option with an exercise price of K_1. Assume that both options share the same expiration date, and $K_2 > K_1$. What type of forward position obtains when both of these options also share the same exercise price (i.e., $K_1 = K_2$)?

SOLUTION:The trading profits and losses which obtain from selling a call option with an exercise price of K_2 and buying a put option with an exercise price of K_1 are shown in the diagram below.

The position formed here is commonly referred to as a "range forward"; when both of these options also share the same exercise price (i.e., when $K_1 = K_2$), then this Date T Portfolio is synthetically equivalent to a short forward.