Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

Options, Futures and Other Derivatives — Name: SOLUTIONS
Dr. Garven
Problem Set 7

Problem 1.
Consider a European call option on a non-dividend-paying stock where the stock price is \$40, the strike price is \$40, the risk-free rate is 4% per year, the volatility is 30% per year, and the time to maturity is six months.

1. *Calculate u, d, and q for a two step tree (note: assume that the tree is "recombining"; thus, $u = e^{\sigma\sqrt{\delta t}}$ and $d = e^{-\sigma\sqrt{\delta t}}$).*
2. *Value the option using a two step tree.*
3. *Verify that the "Cox-Ross-Rubinstein compared with Black-Scholes-Merton spreadsheet" (available from http://fin4366.garven.com/spring2024/CRR-vs-BSM.xlsm) provides the same answer.*
4. *Use the "Cox-Ross-Rubinstein compared with Black-Scholes-Merton spreadsheet" (available from http://fin4366.garven.com/spring2024/CRR-vs-BSM.xlsm) to value the option with 5, 50, 100, and 500 time steps.*

(1) In this case $\delta t = 0.25$ so that $u = e^{0.30\times\sqrt{0.25}} = 1.1618$, $d = 1/u = 0.8607$, and

$$q = \frac{e^{0.04\times 0.25} - 0.8607}{1.1618 - 0.8607} = 0.4959$$

(2) and (3) The value of the option using a two-step tree as given by the "Cox-Ross-Rubinstein compared with Black-Scholes-Merton spreadsheet" is 3.3739.

(4) With 5, 50, 100, and 500 time steps the value of the option is 3.9229, 3.7394, 3.7478, and 3.7545, respectively.

Problem 2. Consider a European call option on a non-dividend-paying stock where the stock price is \$100, the strike price is \$120, the (annualized) risk-free rate is 2%, $u = e^{\sigma\sqrt{\delta t}}$, $d = 1/u$, the (annualized) volatility is 20% per year, and the time to expiration is six months.

1. Use a 15 timestep tree to determine the current market value of this six-month European call option. Do this manually by applying the Cox-Ross-Rubinstein framework, and be sure to show your work.

   The CRR call option pricing formula for 15 timesteps is:

$$c = e^{-.02(.5)}\left[\sum_{j=a}^{15}\left(\frac{15!}{j!\,(15-j)!}\right)q^j(1-q)^{15-j}(u^j d^{15-j}S - K)\right],$$

where a is the smallest integer greater than $b = \ln(K/Sd^n)/\ln(u/d)$. Since $u = e^{\sigma\sqrt{\delta t}} = e^{.20\sqrt{.5/15}}$ $= 1.0372$, $d = 1/u = 1/1.0372 = 0.9641$, and $b = \ln(120/100(0.9641^{15}))/\ln(1.0372/0.9641) =$ 9.9965, then $a = 10$. Therefore, this option will be in the money at nodes where there are 10 and more up moves. Furthermore, $q = \dfrac{e^{r\delta t} - d}{u - d} = \dfrac{e^{.02(.5)} - .9641}{1.0372 - .9641} = .5$. Thus,

$$c = e^{-.02(.5)}\left[\sum_{j=10}^{15}\left(\tfrac{15!}{j!(15-j)!}\right).5^j(1-.5)^{15-j}(1.0372^j.9641^{15-j}100 - 120)\right]$$

$$= .99\begin{bmatrix} 3,003(0.00003052)(120.03-120) + 1,365(0.00003052)(129.12-120) \\ 455(0.00003052)(138.91-120) + 105(0.00003052)(149.93-120)+ \\ 15(0.00003052)(160.75-120) + 1(0.00003052)(172.93-120) \end{bmatrix}$$

$$= \$0.75$$

2. What is the current market value of an otherwise identical (i.e., same underlying asset, same strike price, interest rate, same volatility, same number of timesteps, and time to expiration) European put option?

   Applying put-call parity, $p = c + Ke^{-rT} - S = \$0.75 + \$120e^{-.02(.5)} - \$100 = \$19.56$.