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Options, Futures and Other Derivatives — Name: SOLUTIONS
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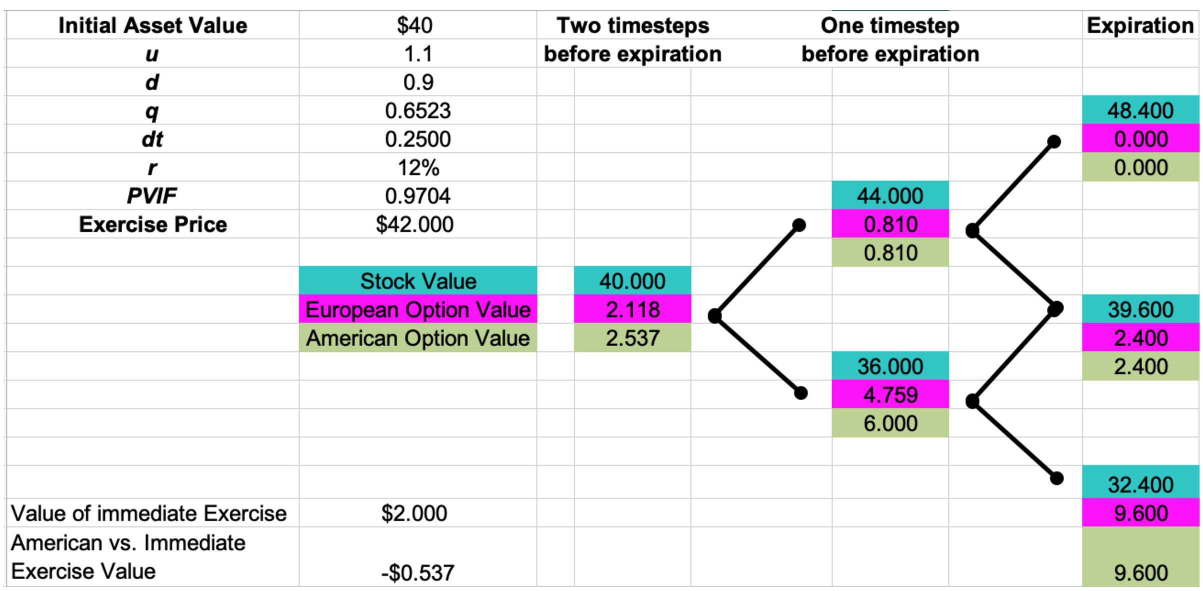
Problem Set 8

Problem 1.
A stock price is currently \$40. Over each of the next two three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per year with continuous compounding.

A. *What is the value of a six-month European put option with a exercise price of \$42?*

B. *What is the value of a six-month American put option with a exercise price of \$42?*

A. A tree describing the behavior of the prices for the stock, the European put, and the American put is shown in the following figure - produced using the worksheet labeled "American vs. European Put" in the spreadsheet located at http://fin4366.garven.com/spring2024/trial_error.xls:



Tree to evaluate European and American put options in Problem 1. At each node, the upper number is the stock price, the next number is the European put price, and the final number is the American put price.

The risk-neutral probability of an up move, q, is given by

$$q = \frac{e^{0.12\times 3/12} - 0.90}{1.1 - 0.9} = 0.6523$$

Calculating the expected payoff and discounting, we obtain the value of the option as

$$[2.4 \times 2 \times 0.6523 \times 0.3477 + 9.6 \times 0.3477^2]e^{-0.12\times 6/12} = 2.118$$

The value of the European option is 2.118. As shown in the figure above, this can be calculated by working back through the tree. The second number at each node is the value of the European option.

B. The value of the American put option is shown as the third number at each node on the tree. At the tree's inception, the American put is worth \$2.537. This exceeds the value of the European put option because it is optimal to exercise early at the d node. However, it would not be worthwhile to exercise the American put immediately since it is only worth $K - S =$ \$2 at the tree's inception.

Problem 2.
Using a "trial-and-error" approach, estimate how high the exercise price has to be in Problem 1 for it to be optimal to exercise the option immediately.

At a \$42 exercise price, it would not be worthwhile to exercise the American put option immediately since its immediate exercise value is \$2, whereas its value as a "live" option (to be exercised early by the investor at node d) is \$2.537. Consequently, the exercise price must be greater than \$42 for it to be worthwhile to exercise immediately.

My "trial and error" solution is provided by using Solver in the spreadsheet reference in Problem 1:

Initial Asset Value	$40	**Two timesteps before expiration**	**One timestep before expiration**	**Expiration**
u	1.1			
d	0.9			
q	0.6523			48.400
dt	0.2500			0.000
r	12%			0.000
PVIF	0.9704		44.000	
Exercise Price	$43.197		1.214	
			1.214	
	Stock Value	40.000		
	European Option Value	2.766		39.600
	American Option Value	3.197		3.597
			36.000	3.597
			5.920	
			7.197	
				32.400
Value of immediate Exercise	$3.197			10.797
American vs. Immediate Exercise Value	$0.000			10.797

By increasing the exercise price (done by having Solver set cell B19 to a value of zero by changing cell B8), the value of immediate exercise increases to \$43.197, a price at which one would be *indifferent* about immediately exercising versus not exercising. Thus, any exercise price above \$43.197 will make it preferable to exercise immediately.

This result can also be analytically shown to be true. Suppose the exercise price increases by a relatively small amount; say by $\$x$. This change will also increase the value of being at node d by $\$x$ since one would exercise the American put at this node even if $x = 0$. The value of being at node u will increase by the (risk neutral probability adjusted) present value of x; i.e., by the amount $(1-q)e^{-r\delta t}x = 0.3477e^{-0.03}x = 0.3374x$. Thus, an increase in the exercise price in the amount of x increases the value of owning the American put at the option's inception by the following amount

$$\Delta p = e^{-r\delta t}[q\Delta p_u + (1-q)\Delta p_d] = e^{-0.03}(0.6523 \times 0.3374x + 0.3477x) = 0.551x.$$

For *immediate* early exercise to be optimal, the sum of the American put value when the exercise price is \$40 (\$2.537) plus the increase in option value from increasing the exercise price by $\$x$ ($\$0.551x$) must be *less than* the sum of the payoff from immediate exercise when the exercise price is \$40 (\$2) plus the increase in the exercise price ($\$x$); i.e.,

$$2.537 + 0.551x < 2 + x \Rightarrow 0.537 < 0.449x \Rightarrow x > 1.197.$$

This corresponds to the exercise price being greater than \$43.197.